TITLE: Linear Matrix Equation Solver and General Matrix Inversion Using Drum Storage   (SADOI Only)

TYPE: Closed subroutine with one S parameter.

TEMPORARY STORAGE: 0 thru 8

NUMBER OF WORDS: 256

DESCRIPTION: This routine is a subroutine version of M 24. It solves the matrix equation AX = B given A and B where

1. A is a non-singular square matrix of order n.

2. B is of size (n x m). B may be the identity matrix I of size (n x m).

The magnitudes of n and m are governed by the storage available in the Williams memory and on the drum. The matrix uses

$$2n + 4m + 1$$

storage locations in the Williams memory and a minimum of

$$\frac{n(n + 1)}{2} + n(m + 1) , \qquad (B = I)$$

or $\qquad n(n + m + 2) , \qquad\qquad (B \neq I)$

storage locations on the drum.

The subroutine takes the matrices (or matrix) from the drum and puts the solution back on the drum.

S PARAMETER: The parameter S3 is used to determine the first location, S, of Williams memory available for storage. It then uses the $2n + 4m + 1$ successive locations starting with S. The parameter has the form

    00 F
    00 SF

METHOD OF USE: This subroutine assumes the equations to be solved are on the drum, beginning at location d_0, stored by row with a sum check after each row, in the form

$$a_{11}, \; a_{12}, \; a_{13}, \; \cdots, \; a_{1n} , \qquad \text{SUM CHECK}$$
$$b_{11}, \; b_{12}, \; \cdots, \; b_{1m} , \qquad \text{SUM CHECK}$$
$$\vdots \qquad\qquad\qquad\qquad \vdots$$
$$a_{n1}, \; a_{n2}, \; a_{n3}, \; \cdots, \; a_{nn} , \qquad \text{SUM CHECK}$$
$$b_{n1}, \; b_{n2}, \; \cdots, \; b_{nm} , \qquad \text{SUM CHECK}$$

where a_{ij} and b_{ij} are the elements of the matrices A and B. In the case of inversion B is not placed on the drum and the successive rows of A follow one another immediately.

The routine uses $\frac{n(n + 1)}{2} + n(m + 1)$ drum locations for temporary storage starting with location d_1.

The solution is stored by row with a sum check after each row starting with drum location d_2. The scaling factors are as computed in M 24 and are stored as an extra row of the solution matrix. This takes $(n + 1)(m + 1)$ locations.

Before entry to this subroutine, Williams memory locations 3 through 7 must contain

3: n,   order of matrix A

4: m,   number of columns in matrix B (m = n
       for inversion, B = I)

5: d_0, as defined above

6: d_1, "   "   "

7: d_2, "   "   "

Entry is of the form

       p 50 F

          50 pF
       _____

          26 (M27)

for the solution of the linear matrix equation, and

       p JO F

          50 pF
       _____

          26 (M27)

for inversion. Control is returned to the right side of location p + 1.

If a solution has been obtained, location 0 contains zero. If A is singular, hence no solution, location 0 is set to 1/2.

DURATION:

Characteristic times are

       n, m = 6   7 seconds

       n, m = 10   21 seconds

       n, m = 15   54 seconds

For large n, the time goes up as n^3. Due to the large use of the drum the routine should only be used for n > 20. M 14 may be used for lower order matrices.

NOTE:

1. Subroutine Y 1 is contained in the subroutine itself and may be used separately. It has the symbolic address (Y 1).

2. In the case of inversion, care should be taken to keep d_0 and d_1 far enough apart. One must have

$$d_1 \leq d_0 - [n(m + 1) - \frac{n(n + 1)}{2}]$$

or $\qquad d_1 > d_0 + n(n + 1).$

Suitable choices of d_0, d_1, and d_2 will allow one to have the original matrices available <u>after</u> obtaining a solution.
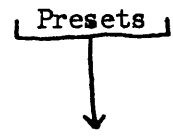
3. A stop with

FF 010
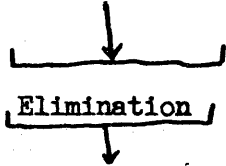
42 ---

signifies a sum check stop in Y 1. A white switch start will attempt to read again.

DATE___May 26, 1959_____

PROGRAMMED BY _S J Penny_

APPROVED BY_____

lgr

LOCATION	ORDER	NOTES
	00 K(M27)	
0	K5 F	Link
	42 204L	
1	40 8F	I
	L5 5F	
2	L4 215L	
	40 29L	d_0
3	L5 6F	
	L4 **215L**	
4	40 183L	d_1
	L5 7F	
5	L4 215L	
	40 190L	d_2
6	L5 3F	
	00 20F	
7	46 30L	n
	L5 4F	
8	50 205L	
	00 20F	
9	46 42L	m
	46 191L	
10	46 202L	
	L4 30L	
11	46 210L	$\mu = n + m$
	46 184L	
12	L5 210L	
	L4 3F	
13	42 122L	$y = S3 + n$
	42 143L	
14	00 20F	
	46 40L	
15	46 122L	
	46 137L	
16	46 176L	
	46 189L	

Presets

LOCATION	ORDER	NOTES
17	L5 122L	
	L4 4F	
18	42 60L	$t = y + m = S3 + \mu$
	42 63L	
19	42 66L	
	42 211L	
20	42 214L	
	00 20F	
21	46 56L	
	46 95L	
22	46 131L	
	46 162L	
23	46 212L	
	L4 210L	
24	46 188L	$u = S3 + 2\mu$
	L4 42L	
25	46 200L	$v = S3 + 3\mu$
	L0 42L	
26	10 20F	
	42 213L	u
27	41 6F	$r \to 0$
	41 7F	$i \to 0$
28	50 S3	
	50 28L	
29	26 (Y1)	
	00 (d$_0$)F	Call row r of A from Drum
30	00 (n)F	
	F5 29L	
31	L4 3F	
	40 29L	
32	40 41L	
	L5 8F	
33	36 40L	B $\neq$ I?
	L5 122L	
34	42 35L	
	L4 6F	

Elimination

(braces at 29–30): 2', 31', 43', 7

LOCATION	ORDER		NOTES
35	42 38L		
	41 (y')F	34	Augment A with B = I
36	F5 35L		
	42 35L		
37	L0 214L		
	32 35L		
38	L5 209L		
	40 (y + r)F	35	
39	26 44L		
	00 F		Waste
40	50 (y)F	14'	
	50 40L		
41	26 (Y1)	32	Call row r of B from Drum
	00 (d)F		
42	00 (m)F		
	F5 41L		
43	L4 4F		
	40 29L		
44	L5 183L		
	40 57L		$d = d_1$
45	40 96L		Preset Drum Commands
	L5 210L		
46	46 58L		$v = \mu$
	46 97L		
47	L5 210L		
	L4 7F		$x = S3 + 1$
48	42 64L		
	42 69L		
49	42 73L		
	42 89L		
50	42 91L		Set x addresses
	42 114L		
51	00 20F		
	46 61L		
52	46 63L		
	46 67L		

LOCATION	ORDER		NOTES
53	46 73L		
	46 75L		
54	46 113L		
	L3 6F		
55	L6 7F		$i \geq m'$
	36 106L		
56	50 (t)F	21	
	50 56L		
57	26 (Y1)	44'	Call row i for
	00 (d)F	99'	elimination
58	00 (v)F	46 101'	
	26 59L		
59	L5 206L		Reset interchange
	46 70L		
60	46 71L		
	L3 (t)F	18	
61	L6 (x)F	51'	$\lvert x \rvert \geq \lvert t \rvert$?
	32 64L		
62	47 70L		$L_1 \to 0$
	50 205L		
63	L5 (x)F	52	$k = \dfrac{x}{t}$
	66 (t)F	18'	
64	26 68L		
	50 (x)F	48	
65	S3 F		$x = 0$?
	32 67L		
66	50 208L		$k = \dfrac{t}{x}$
	75 (t)F	19	
67	66 (x)F	52'	
	47 71L		$L_2 \to 0$
68	41 5F		Clear M box
	S1 F		Store − k
69	40 2F		
	L5 (x)F	48' 80	
70	40 (L1)F	59' 62	Interchange
	L5 (t)F	102' 78	

LOCATION	ORDER		NOTES
71	40 (L₂)F	60 67'	
	50 2F		
72	7J 1F		$x_j' = x_j - kt_j, \ k = \dfrac{x_1}{t_1}$ or
	L4 F		$t_j - kx_j, \ k = \dfrac{t_1}{x_1}$
73	40 (x)F	53⎫ 79'	
	L3 (x)F	49⎭	
74	L6 5F		$|M| > |x_j'|$
	36 76L		
75	L7 (x)F	53' 80'	Replace $|M|$ by $|x_j'|$
	40 5F		
76	L5 1F		
	40 (t)F	77' 103	Complete change
77	F5 76L		
	42 76L		
78	42 70L		
	L5 73L		Advance
79	L4 206L		addresses
	40 73L		
80	42 69L		
	46 75L		
81	L0 212L		
	32 69L		More elements?
82	L5 207L		
	40 90L		
83	LL 5F		
	32 85L		
84	L5 69L		
	46 90L		
85	26 89L		
	L5 90L		
86	42 90L		
	L5 5F		
87	00 1F		
	40 5F		
88	LL 5F		Scale to
	32 85L		$\dfrac{1}{2} < |a|_{Max} \leq \dfrac{1}{4}$

LOCATION	ORDER	NOTES	
89	50 205L		
	L5 (x)F	49' 93	
90	10 (1)F	84' ⎫ 82'	
	00 (1)F	86 ⎭	
91	50 205L		
	40 (x)F	50 92'	
92	F5 91L		
	42 91L		
93	42 89L		
	L0 211L		
94	32 89L		
	26 95L		
95	J0 (t)F	21' 104 110'	
	50 95L		
96	26 (Y1)	45 ⎫	
	00 (d)F	99 ⎭	
97	00 (v)F	46' 101	
	L5 97L		Put row i back on drum
98	10 20F		
	F4 96L		
99	40 96L		
	40 57L		
100	L5 97L		
	L0 206L		
101	46 97L		
	46 58L		
102	L5 211L		
	42 70L		
103	42 76L		Reset t addresses
	L5 212L		
104	46 95L		
	F5 7F		
105	42 7F		$i \rightarrow i + 1$
	26 47L		
106	L3 7F		
	L6 6F		i = r?

LOCATION	ORDER		NOTES
107	36 110L		
	F5 6F		
108	42 6F		$r \to r + 1$
	L0 3F		$r = M?$
109	36 121L		
	22 27L		
110	L5 113L		
	46 95L		
111	L3 6F		
	32 112L		
112	26 95L		
	41 5F		
113	L3 (x)F	117 54	
	L6 5F		
114	32 115L		
	L7 (x)F	116' 50'	Prepare to put row r
115	40 5F		back on Drum
	F5 114L		
116	42 114L		
	00 20F		
117	46 113L		
	F5 7F		
118	42 7F		
	L0 3F		
119	L0 4F		
	36 82L		
120	26 113L		
	00 F		
121	41 5F		Clear k
	L5 209L		$S \to S_0 = \dfrac{1}{10}$
122	40 (y)F	15'	Stores
	L3 (y)F	13	Sing? $S < 2^{-39}$
123	32 203L		
	L5 96L		
124	40 132L		Set Drum Addresses
	L5 97L		

Back Substitution

LOCATION	ORDER		NOTES
125	46 133L		
	41 6F		
126	41 7F		Clear Counters
	L5 132L		
127	F0 4F		
	F0 6F		
128	40 132L		Adjust Drum Addresses
	40 163L		
129	L5 133L		
	L4 206L		
130	46 133L		
	46 164L		
131	50 (t)F	22	
	50 131L		
132	26 (Y1)	124	Call row (n - r) from Drum
	00 (d)F	128	
133	00 (v)F	125 130	
	L5 211L		
134	F4 6F		
	42 140L		
135	L4 5F		Reset addresses
	42 137L		
136	L5 137L		
	46 140L		
137	50 (y)F	15'	
	71 (b)F	135'	
138	40 F		
	22 144L		
139	00 F		Compute
	S5 F		$\sum\limits_{r=1}^{n} a_{1r} x_r$
140	50 (x)F	136' } 145'	
	74 (a)F	134'	
141	L4 F		
	40 F		
142	LL F		$\lvert \Sigma \rvert < \frac{1}{2}$?
	32 144L		

LOCATION	ORDER		NOTES
143	50 209L		
	7J (y)F	13'	Rescale
144	26 122L		
	L5 140L		
145	L0 206L		
	40 140L		
146	42 150L		
	42 153L		Advance addresses
147	46 155L		$(n - r + 1)$ times
	F5 7F		for row r
148	42 7F		
	L5 6F		
149	L0 7F		
	32 139L		
150	41 7F		Reset i
	L3 (a_{11})F	146	Sing? $a_{ii} = 0$
151	32 203L		
	L6 F		Division bad?
152	36 143L		
	26 153L		
153	L5 F		
	66 (a_{11})F	146'	$x_r = \dfrac{\Sigma}{a_{rr}}$
154	22 154L		
	S1 F		
155	40 (x)F	147	
	L3 5F		k = 0?
156	32 164L		
	L5 155L		
157	L4 210L		
	L4 210L		
158	46 161L		Restore row $(n - r)$
	L5 211L		of augmented matrix
159	L4 5F		to Drum with
	L4 6F		$x_{n-1, r}$
160	42 161L		
	26 161L		

LOCATION	ORDER		NOTES
161	L5 (u*)F	158	
	40 (∝)F	160	
162	J0 (t)F	22'	
	50 162L		
163	26 (Y1)	} 128'	
	00 (d)F		
164	00 (v)F	130'	
	F5 6F		
165	42 6F		
	L0 3F		Count n rows
166	36 167L		
	22 126L		
167	L5 213L		
	40 169L		
168	41 F		
	26 169L		
169	L5 S3	} 167'	
	40 (u)F	} 171	
170	L5 169L		
	L4 206L		
171	40 169L		
	F5 F		Store column
172	42 F		k for later use
	L0 4F		
173	36 174L		
	26 169L		
174	L5 200L		
	10 20F		
175	L4 5F		
	42 176L		
176	L5 (y)F	16	
	40 (v')F	175'	
177	F5 5F		
	42 5F		k → k + 1
178	L0 4F		
	32 179L		k ≠ m?

LOCATION	ORDER	NOTES		
179	22 121L			
	L5 211L			
180	L0 206L			
	42 188L			
181	41 2F			
	26 182L			
182	50 S3			
	50 182L			
183	26 (Y1)	} 4		
	00 (d_1)F	} 186		
184	00 (v)F	11' 187'		
	L5 184L			
185	10 20F			
	F4 183L			
186	40 183L			
	L5 184L			
187	L0 206L	Arrange X on Drum		
	46 184L	by rows		
188	L5 (u)F	24 196		
	40 (t')F	180' 197		
189	J0 (y)F	16' 192'		
	50 189L			
190	26 (Y1)	} 5'		
	00 (d)F	} 194		
191	00 (m)F	9'		
	L5 189L			
192	L0 206L			
	46 189L			
193	L5 190L			
	F4 4F			
194	40 190L			
	40 201L			
195	L5 188L			
	L4 206L			
196	46 188L			
	L0 205L			

LOCATION	ORDER		NOTES
197	42 188L		
	F5 2F		
198	42 2F		
	LO 3F		Count n rows
199	36 200L		
	26 182L'		
200	JO (v)F	25	
	50 200L		
201	26 (Y1)	} 194'	Store scale constants as
	00 (d)F		row n + 1 of x
202	00 (m)F	10	
	41 F		0 if completed
203	22 204L		
	49 F		½ if singular
204	22 204L		
	22 (Link)F	0'	Exit
205	00 F		
	00 2F		
206	00 1F		
	00 1F		
207	10 1F		
	00 1F		
208	7L 4095F		
	LL 4095F		
209	00 F		
	00 1000 0000 0000J		Constants
210	00 (μ)F	11	
	00 S3		
211	JO 205L		
	40 (t)F	19'	
212	NO (t)F	23	
	L3 F		
213	L5 S3		
	40 (u)F	26'	

LOCATION	ORDER	NOTES		
214	M2 38L			
	41 (t)F	20		
215	26 (Y1)			
	00 F			
(Y1)	00 K		Y 1 - 199 Transfer Blocks of Words from the Memory to the Drum or from the Drum to the Memory	
	24 999N			