

UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

LIBRARY ROUTINE M 17 - 197

TITLE Post Multiplication of a Matrix A or Its Transpose
 A^T by a Vector k.

TYPE Closed with one program parameter. Used with N - 8.
The link is:

P	AO ℓ F	;
	50 pF	
	26 xF	

where ℓ is the location beginning at which non-zero elements of the matrix A are stored and x is the first word of this routine. Set A = K for Ak; A = 5 for $A^T k$,

NUMBER OF WORDS 45

TEMPORARY STORAGE 0,1

PARAMETER S3, S4, S5. During the input of this routine locations 3, 4 and 5 must hold the following quantities:

- 3) 00 F 00 nF n is the number of rows in the matrix A.
- 4) 00 F 00 bF Location (b + i), (i = 0, ..., q-1), contain the q components of the vector k.
- 5) 00 F 00 cF Locations (c + i)(i = 0, ..., q-1) contain the q components of the vector product Ak or $A^T k$.

ACCURACY A maximum of 9 decimal places

DURATION

$$\sum_{i=1}^r (1.53 c_i + .98) \text{ ms for Ak,}$$

$$\sum_{i=1}^r (1.59 c_i + .80) \text{ ms for } A^T k,$$

where

c_i = number of non-zero elements in row r_i
 r = number of rows of A.

DESCRIPTION This routine will postmultiply any matrix A or its transpose A^T by a column vector k. It is designed to handle a matrix where only the non-zero elements are stored. These non-zero elements are assumed to be stored consecutively. Each non-zero element utilizes the last ten bits to indicate the row and column in which it occurred. The first non-zero element of each row has one in the 2^{-30} th location; and the other non-zero elements of the same row have zero in the 2^{-30} th location. The remaining nine bits 2^{-31} , ..., 2^{-39} indicate the column index. The last non-zero element of A must be followed by a word containing 2^{-30} . Routine N - 8 will input and store the non-zero elements of a matrix properly modified.

LIMITATIONS This routine will not handle any matrix with more than 511 columns. The elements a_{ij} of the matrix A and the elements k_j of the vector k must be scaled so that

$$\sum_{j=1}^n |A_{ij} k_j| < 1 \text{ in the case of } Ak,$$

and

$$\sum_{i=1}^n |A_{ij}^T k_i| < 1 \text{ in the case of } A^T k.$$

This will be true if

$$|k_j| < x/n \text{ with } x^2 < n$$

and

$$|a_{ij}| < x/m \text{ in the case of } Ak, \text{ or}$$
$$|a_{ij}^T| < x/m' \text{ in the case of } A^T k,$$

where

n = number of components of the vector k

$m = \max (r_1, r_2, \dots, r_p, \dots)$

where r_i is the number of non-zero elements in the i th row.

$m' = \max (c_1, c_2, \dots, c_q, \dots)$

where c_i is the number of non-zero elements in the i th column.

It is advisable to scale down by powers of 10 so that one can count back merely by shifting the decimal point.

NOTES

1) If one wishes to perform only A_k type multiplications, he may delete the last 20 words of this routine, and thus reduce it in length to 25 words.

2) If one wishes to use this routine repeatedly he should clear out the locations $(c + i)$ between entries. Otherwise, the components of the previous multiplications A_{1k_1} will have been added to the respective components of the present multiplication A_{2k_2} with the final result being $A_{1k_1} + A_{2k_2}$ instead of A_{2k_2} .

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APPROVED BY J. P. Nash

RPF/mge

11/9/55

LOCATION	ORDER		NOTES	PAGE 1	M 17
0	K5 F				
	42 12L				
1	36 25L		to A ^T _k		
	10 20F		Set addresses for finding		
2	42 17L		elements of A		
	42 20L				
3	41 1F				
	F1 1F		Set counter to -1		
4	40 1F				
	L4 5F		Set address for		
5	42 15L		storing A _k components		
	00 20F				
6	46 15L				
	22 17L				
7	40 F				
	50 F				
8	F5 15L				
	40 15L		New Row		
9	L5 11L		reset addresses,		
	L4 15L		increase counter		
10	46 15L		and test for end		
	26 11L		waste		
11	F5 1F				
	40 1F				
12	L0 3F				
	32 ()F				
13	S5 F				
	10 24L				
14	26 20L				
	7J F				
15	L4 F				
	40 F				
16	F5 17L				
	40 17L				

LOCATION	ORDER	NOTES	PAGE 2	M 17
17	42 20L 50 ()F	Form Ak		
18	J0 23L S5 F			
19	L0 24L 36 7L	Test for new row		
20	L4 22L 50 F			
21	42 14L 22 14L			
22	00 F 00 1535S4	(1023 +512) S4		
23	00 F 00 1023F	Constants		
24	00 F 00 512F			
25	10 20F 42 38L	Set addresses for finding elements of A^T		
26	42 42L 41 1F	Set counter		
27	F1 1F 40 1F	= -1		
28	L4 4F 00 20F	Set for address of		
29	46 43L 22 38L	1st component of k		
30	40 F 50 F			
31	L5 43L L4 36L	New Row, reset		
32	46 43L F5 1F	addresses, increase counter, and test		
33	40 1F L0 3F	for end		

LOCATIONS	ORDER		NOTES	PAGE 3
34	32 12L			
	S5 F			
35	L0 24L			
	26 41L			
36	00 F		waste	
	40 ()F			
37	F5 38L			
	40 38L			
38	42 42L			
	50 ()F		Form A ^T k	
39	J0 23L			
	S5 F			
40	L0 24L			
	36 30L			
41	L4 44L			
	42 43L			
42	42 36L			
	50 ()F			
43	7J ()F			
	L4 ()F			
44	22 36L			
	00 1535S5		(1023 + 512) S5	

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11/7/55