TITLE	Autocorrelations
TYPE	Entire Program
DURATION	See below
METHOD OF USE	The program is read into the memory in the usual way.

After the master tape is read in a sum check is performed. If the master tape has been read in incorrectly, ten sexadecimal characters will be punched. The master tape should then be read into the memory again. Next, the data tape is placed in the reader and read in. The first character that appears on the print-out gives the type of autocorrelation. This is followed by the mean of the observations and then the variance. Two columns appear next. The first column is the lag and the second is the autocorrelation. After all the autocorrelations have been computed (the lags having increments of unity from first to last), a new problem can be begun.

CAPACITY The maximum number of observations this program will handle is 795.

PUNCHING OF TAPES Each observation which must be in the range $-1 \leq x < 1$ is punched as a sign followed by up to 12 decimal digits. The final fraction in the sequence must be followed by an N, J F, or L. Each character signifies a different type of autocorrelation. These are given by the following:

N:
$$\frac{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})(x_{i+h} - \bar{x})}{s^2},$$

with $x_j = x_{j-n}$ if $j > n$,

where $s^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$;

$$J: \quad \dfrac{\dfrac{1}{n-h} \displaystyle\sum_{i=1}^{n-h} (x_i - \bar{x}_1)(x_{i+h} - \bar{x}_2)}{s^2}$$

where $\bar{x}_1 = \dfrac{1}{n-h} \displaystyle\sum_{i=1}^{n-h} x_i$,

and $\bar{x}_2 = \dfrac{1}{n-h} \displaystyle\sum_{i=1}^{n-h} x_{i+h}$;

$$F: \quad \dfrac{1}{n-h} \displaystyle\sum_{i=1}^{n-h} \dfrac{(x_i - \bar{x})(x_{i+n} - \bar{x})}{s^2}$$

$$L: \quad \dfrac{1}{n-h} \displaystyle\sum_{i=1}^{n-h} \dfrac{(x_i - \bar{x}_1)(x_{i+h} - \bar{x}_2)}{s_1 \, s_2}$$

where $s_1 = \displaystyle\sum_{i=1}^{n-h} \dfrac{(x_i - \bar{x}_1)^2}{n-h}$

and $s_2 = \displaystyle\sum_{i=1}^{n-h} \dfrac{(x_{i+h} - \bar{x}_2)^2}{n-h}$

This is followed by an integer giving the initial lag
desired, a fifth hole character, an integer giving
the last lag desired, another fifth hole character, and
finally by the number of decimal places to be printed.
The number of places to be printed must not exceed 9.
When the problem is completed, a new data may be read
into the computer, a different type of autocorrelation
using the same data may be computed, or the same type
may be computed over a different range of lags. To
use the data in the computer again it is only necessary
to read in new parameters.

Time per Autocorrelation of Lag h

Type	Computation	Output
N	$2.8\,n + 85.3$ milliseconds	$50(p+4)$ milliseconds
J	$4.1(n-h) + 86.9$ "	"
F	$4.1(n-h) + 87.2$ "	"
L	$58.7(n-h) + 220$ "	"

where n = number of observations

and p = number of places each correlation is to be printed.

DATE 4/7/55 Rt: 8/5/55

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LOCATION	ORDER	NOTES
	Routine X 1	Decimal Order Input
	00 166K	
	Routine N 2	Input a Sequence of Decimal Fractions
	00 192K	
	Routine P 1	Print One Number
	00 220K	
	Routine R 1	Square Root Routine
	00 3K	
	00 F	
	00 229F	
	00 F	
	00 166F	
	00 F	
	00 192F	
	00 F	
	00 220F	
	00 25K	
0	92 135F	
	41 5F	
1	50 S3	
	50 1L	Read in Fractions
2	26 S4	
	40 3F	
3	L5 8S4	
	10 20F	
4	42 137L	Test how many numbers have been
	L0 140L	read in
5	40 16F	
	L0 131L	
6	40 17F	n
	19 38F	
7	66 17F	
	S5 S3	
8	40 18F	1/n
	L5 7L	
9	42 10L	
	41 8F	

LOCATION	ORDER		NOTES
10	41 9F		
	L5 F	by 9, 16; from 17	
11	40 1F		
	50 1F		
12	7J 18F		
	40 2F		
13	L4 8F		
	40 8F		Compute mean and sum of squares
14	50 2F		
	7J 1F		
15	L4 9F		
	40 9F		
16	F5 10L		
	42 10L		
17	L0 137L		
	32 10L		
18	50 8F		
	79 8F		
19	L4 9F		
	40 9F		Compute variance
20	L5 3F		
	L4 24L		
21	42 26L		
	00 36F		
22	82 4F		Punch out type of autocorrelation
	L5 3F		
23	F4 23L		
	42 90L		Determine type of autocorrelation desired
24	41 13F		
	41 12F		
25	41 14F		
	41 15F		
26	L5 132L		
	40 F	by 21	
27	L5 133L		
	42 31L		

LOCATION	ORDER		NOTES		
28	50 5F	from 34			
	81 4F				
29	74 133L	from 30			
	00 4F				
30	91 4F				
	36 29L		Read in initial and last lag		
31	S5 F				
	40 F	by 27, 32			
32	F5 31L				
	42 31L				
33	L1 L				
	40 L				
34	36 28L				
	41 6F				
35	81 4F		Read in number of places to be printed		
	00 20F				
36	46 6F				
	10 20F				
37	L4 6F				
	10 39F				
38	L4 6F				
	74 133L		Compute print routine parameters		
39	K5 F				
	46 45L				
40	46 48L				
	00 20F				
41	46 118L				
	92 135F				
42	F5 11F				
	L4 131L				
43	40 11F				
	92 515F				
44	92 987F				
	L5 8F				
45	52 F	by 39			
	50 45L		Print out mean		

LOCATION	ORDER		NOTES PAGE 4 K 5
46	26 S5		
	92 131F		
47	92 987F		
	L5 9F		
48	52 F	by 40	
	50 48L		Print out variance
49	26 S5		
	L5 134L		1/10
50	40 6F		
	50 18F	from 121	
51	F1 12F		
	32 55L		
52	L5 10F		
	L0 17F		
53	40 0F		
	09 1F		
54	10 39F		
	66 0F		
55	L5 10F		
	40 7F		
56	S5 F		
	40 19F		Compute 1/n-h or 1/n
57	41 20F		
	41 21F		
58	41 22F		
	41 23F		
59	41 24F		
	92 131F		
60	L5 137L		
	42 136L		
61	92 515F		
	L5 135L		
62	42 63L		
	L4 10F		
63	42 65L		
	L5 F	by 62, 81; from 87	

LOCATION	ORDER		NOTES
64	10 1F		$x_i/2$
	40 0F		
65	32 65L	by 63, 83	
	L5 F		
66	10 1F		$x_{i+h}/2$
	40 1F		
67	50 19F		
	7J 0F		
68	40 2F		
	50 2F		
69	7J 1F		
	L4 21F		
70	40 21F		$\dfrac{\sum x_i\, x_{i+h}}{4.(n - \delta h)}$ where $\delta = 0$ for r circular
	F1 12F		$= 1$ for r non-circular
71	32 80L		
	L5 2F		
72	L4 22F		$\dfrac{\sum x_i}{2(n-h)}$
	40 22F		
73	50 19F		
	7J 1F		
74	40 3F		
	L4 23F		
75	40 23F		$\sum x_{i+h} / 2(n-h)$
	L3 15F		
76	32 80L		$\sum x_i^2 / 2(n-h)$
	50 2F		
77	7J 0F		
	L4 24F		
78	40 24F		
	50 3F		
79	7J 1F		
	L4 20F		$\sum x_{i+h}^2 / 2(n-h)$
80	40 20F		
	F5 63L		
81	42 63L		
	F5 65L		

LOCATION	ORDER	NOTES	
82	LO 136L		
	32 84L		Test is $j > n$
83	L4 136L		
	42 65L		
84	22 85L		
	L5 135L	from 82	
85	42 65L		
	F5 7F	from 84	
86	40 7F		
	LO 17F		
87	32 63L		Are computations complete?
	L5 9F		
88	10 2F		
	40 7F		Store $s^2/4$ in 7
89	L5 8F		
	10 1F		Store $\bar{x}/2$ in 4
90	40 4F		
	26 F	by 23; from 130	Determine type of correlation
91	50 4F		
	22 102L		
92	50 22F	from 108, 102	
	22 108L		
93	L5 22F		
	22 103L		
94	50 22F		
	79 22F		
95	L4 24F		
	40 24F		
96	50 23F		
	79 23F		
97	L4 20F		
	40 20F		
98	50 20F		
	75 24F		
99	40 1F		
	S5 F		

LOCATION	ORDER		NOTES		
100	40 OF				
	50 100L				
101	22 S6				
	40 7F				
102	26 92L				
	79 4F	from 91			
103	26 109L				
	L0 4F	from 93			
104	40 OF				
	L5 23F				
105	L0 4F				
	40 1F				
106	50 OF				
	7J 1F				
107	L4 21F				
	40 21F				
108	26 92L				
	79 23F	from 92			
109	L4 21F	from 103			
	40 OF				
110	50 OF				
	75 6F				
111	40 OF				
	L7 F				
112	L2 7F				
	36 127L		Test for scaling		
113	L5 OF				
	66 7F				
114	S5 1F				
	40 4F				
115	32 115L				
	L5 10F		Print out lag		
116	50 44F				
	50 116L				
117	26 S5				
	L5 4F				

LOCATION	ORDER	NOTES	PAGE 8	K 5
118	52 F	by 41		
	50 118L		Print out autocorrelation	
119	26 S5			
	F5 10F			
120	40 10F			
	L0 11F			
121	32 50L			
	24 122L			
122	92 135F			
	81 4F			
123	L0 138L			
	40 3F			
124	36 20L		Test if new data or new parameters	
	L5 139		have been read	
125	40 8S4			
	L5 3F			
126	L4 23S4			
	22 10S4			
127	50 6F	from 112		
	7J 134L			
128	40 6F			
	L5 118L			
129	L4 114L			
	46 118L			
130	22 90L			
	00 F			
131	80 F			
	00 F			
132	LL 4095F			
	LL 4095F			
133	00 F			
	00 10F			
134	00 F			
	00 1000 0000 0000J			
135	00 S3			
	00 S3			

LOCATION	ORDER	NOTES	PAGE 9	K 5
136	32 65L			
	L5 F			
137	N1 9F			
	L5 F	by 4		
138	00 F			
	00 12F			
139	40 S3			
	L5 8S4			
140	80 F			
	40 S3			
	00 500K			
	Routine X 7	Sum Check		
	24 25N			