TITLE   Interpolation   (DOI or SADOI)

TYPE   Closed

NUMBER OF WORDS   51

PARAMETERS   Parameters must be read into locations 3 and 4 before the program is read into the store. A number of program parameters also need to be set.

TEMPORARY STORAGE   0, 1, 2, 3, 4, 5, 6, 7 plus d locations starting at the address entered in S before read-in.

ACCURACY   A maximum rounding-off error of $(2^d - 1) \times 2^{-39}$ where d = depth of interpolation.

DURATION $(6 + 3.5d + 2.4d^2)$ ms where d = depth of interpolation.

DESCRIPTION   The routine interpolates to arbitrary depth, d, in one or more tables which may be stored in the machine. It is assumed that the table consists of a sequential set of function values, $f(x_i)$ for increasing arguments, x_i, equally spaced by $h = x_{i+1} - x_i > 0$. $f(x_i)$ is to be stored in location $(a + [1/h] x_i)$ where $h = 1/n$, n is an integer. $f(0)$ is stored in location a. (Location a itself need not be filled. e.g., with a = 300, h = .01, the entire table might comprise the 11 entries for x_i = .83, .84, .85, ..., .93 which would be stored in locations 383-393). The arguments of the entries can be translated by an arbitrary amount provided that the argument of any function desired is similarly translated.

The routine will be automatically left whenever two successive interpolates differ by less than e where $e < 2^{-19}$ is entered in S4 before read-in. If it is desired to go to fixed depth, S4 should be set to 0 before read-in. Enter with x in R_1 and the following words in the main program

p	50 dF
	50 pF
p+1	26 qF
	00 aF
p+2	00 F
	00 hJ

where q is the location of this routine. (the routine may be entered with x in

O if control is then transferred to the right hand side of q). The routine will be left with f(x) in both A and Q.

RESTRICTIONS

(1) The depth of interpolation d, and the argument step h, must satisfy $|hd| < 1$.

(2) If x_m and x_p be the arguments at the beginning and end of the table respectively, the routine should not be used to interpolate for an x that lies within:

$[(d-1) h/2]$ of x_m or $[(d-1) h/2]$ of x_p if d is odd.

$[(d-2) h/2]$ of x_m or $[(d) h/2]$ of x_p if d is even.

METHOD Neville's method of successive linear interpolations is used:

$$f(x_i x_{i+1} x_{i+2}, \cdots, x_{i+j-1} x_{i+j}) =$$
$$[f(x_i x_{i+1}, \cdots, x_{i+j-1}) (x_{i+j}-x) - f(x_{i+1} x_{i+2}, \cdots, x_{i+j})$$
$$(x_i -x)] / (x_{i+j} - x_j)$$

This expression is applied successively with $j = 1, 2, 3, \ldots, d$.

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lgr

LOCATION	ORDER		NOTES
	00 K(I1)		
0	40 F		Store argument
	41 6F		Clear 6
1	S5 F		Bring in link
	46 6F		Plant d x 2^{-19}
2	L4 18L		Plant table address
	42 12L		
3	L4 18L		Plant h's address
	42 5L		
4	L4 18L		Plant link address
	42 48L		
5	41 7F		Waste
	L5 (p+2)F	By 3	Call in h to 1 and 4
6	40 1F		
	40 4F		
7	L5 6F		
	LO 49L		
8	10 21F		
	42 7F		
9	L5 F		
	10 19F		Compute initial entry addresses to
10	66 1F		table and plant in the linear
	S5 F		interpolator.
11	10 20F		
	LO 7F		
12	40 2F		
	L4 (p+1)F	By 2	
13	42 22L		
	L4 18L		
14	42 20L		
	L5 47L		
15	42 24L		
	50 2F		
16	75 1F		Compute initial $(x_1 - x)$ and (x_1)
	00 39F		

LOCATION	ORDER		NOTES
17	LO F		in 3 and 5 respectively.
	40 F		
18	40 3F		
	L4 1F		
19	40 5F		
	41 7F		Clear 7 for counter
20	50 3F	From 35	
	75 ( )F	By 14,28	
21	40 2F	and 40	
	S1 F		
22	50 5F		THE LINEAR INTERPOLATOR
	74 ( )F	By 13,30,	
23	LO 2F	and 39	
	66 4F		
24	S5 F		
	40 ( )F	By 15,31	
25	L5 7F	and 39	
	L4 49L		Finished with this order of inter-
26	40 7F		polation
	LO 6F		
27	32 35L		
	L5 20L		
28	L4 18L		
	42 20L		Step addresses in the linear inter-
29	L5 22L		polator
	L4 18L		
30	42 22L		
	L5 24L		
31	L4 18L		
	42 24L		
32	L5 3F		
	L4 1F		Advance $[x_1-x]$ and $(x_{1+j}-x)$ by h,
33	40 3F		and repeat.
	L5 5F		
34	L4 1F		

LOCATION	ORDER		NOTES
	40 5F		
35	26 20L		
	L5 6F	From 27	Have we reached desired depth of
36	L0 49L		interpolation?
	40 6F		
37	L0 49L		
	32 38L		
38	22 47L		
	L5 47L	From 37	Reset addresses in the linear
39	42 22L		interpolator.
	42 24L		
40	L4 18L		
	42 20L		
41	L5 4F		
	L4 1F		Advance $(x_{i+j} - x_i)$ by h
42	40 4F		
	L5 F		Reset $(x_i - x)$ in 3
43	40 3F		
	L4 4F		Compute $(x_{i+j} - x)$ and place in 4
44	40 5F		
	L5 S3		
45	L0 1S3		Test difference of successive
	40 2F		interpolates against e
46	L7 2F		
	L0 50L		
47	32 19L		
	L5 S3	From 37	Put f(x) in A and Q and leave
48	50 S3		
	26 (p+3)F	By 4	
49	00 1F		Unit of count
	00 F		
50	00 F		Error parameter = e
	00 34		