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SUBJECT: TRANSIENT RESPONSE OF JUNCTION TRANSISTORS--I

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Abstract: Laplace transforms are used to solve the diffusion equation for hole flow in the base of a junction transistor. Current transfer functions are derived for a planar, homogeneous base transistor. In normalized form:

Common Base

$$I_c(\lambda) = I_e(\lambda)\,\frac{\gamma_N}{\cosh\sqrt{\lambda - \frac{w^2}{L_p^2}}}$$

Common Emitter

$$I_c(\lambda) = I_b(\lambda)\,\frac{\gamma_N}{\cosh\sqrt{\lambda - \frac{w^2}{L_p^2}} - \gamma_N}$$

Expressions are obtained for the inverses of these functions for steps of input current. In agreement with experimental results, a delay is found in the common base response. This leads to a modification of the rise and fall time equations of Ebers and Moll. The common emitter switching times are found to agree quite well with Ebers and Moll.

The storage times are found by breaking up the solution into its forward and reverse components and using the common base transfer function for each. This gives a storage coefficient involving ω_N, ω_I, β_N, β_I, γ_N, and γ_I which can be put approximately in the form of Ebers and Moll's result. The effect of γ_I on storage time is noted.

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TRANSIENT RESPONSE OF JUNCTION TRANSISTORS--I

INTRODUCTION

The flow of minority carriers in a semiconductor is governed by the same diffusion equation that describes heat flow except that a minority carrier can recombine with a majority carrier. In this paper we shall follow the conventional practice of considering the minority carriers to be holes, and the transistor to be pnp. The equations for electrons can be obtained by replacing p by n and q by -q. The continuity equation[1]

$$-\frac{1}{q}\frac{dI_p(x,t)}{dx} - \frac{p(x,t)}{\tau_p} = \frac{dp(x,t)}{dt} \qquad \left[\frac{1}{cm^3 sec}\right] \tag{1}$$

states that in a slab of unit area and width dx, the number of holes entering minus those leaving per second, minus the number recombining per second equals the rate of growth of the hole density. The transport equation states that the number of holes passing through the slab in a second is D_p, the diffusion constant, times the slope of the hole density.

$$\frac{I_p(x,t)}{q} = -D_p \frac{dp(x,t)}{dx} \qquad \left[\frac{1}{cm^2 sec}\right] \tag{2}$$

We now differentiate (2), substitute this in (1) and obtain the diffusion equation.

$$D_p \frac{d^2p(x,t)}{dx^2} - \frac{p(x,t)}{\tau_p} = \frac{dp(x,t)}{dt} \tag{3}$$

This is a one-dimensional diffusion equation in which all hole flow is considered to be parallel and the emitter and collector to be planar and parallel. The assumption here is somewhat like neglecting fringing in a parallel plate capacitor. To solve this partial differential equation we take the Laplace transform with respect to time.

$$D_p \frac{d^2P(s,x)}{dx^2} - \frac{P(s,x)}{\tau_p} = s\,P(s,x) - p(x,0) \qquad (4)$$

To facilitate solution $p(x,0)$ is set to zero, which means that the solution will start from rest and rise to the final value. Equation (4) may be put in the form:

$$\frac{d^2P(s,x)}{dx^2} - \Gamma^2 P(s,x) = 0 \qquad (5)$$

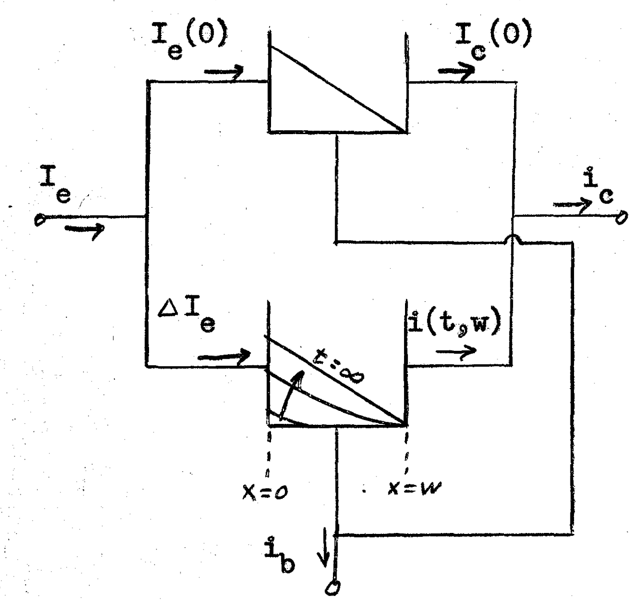
$$\text{where } \Gamma^2 = \frac{s + \frac{1}{\tau_p}}{D_p}$$

with solution:

$$P(s,x) = A \operatorname{Sinh} \Gamma x + B \operatorname{Cosh} \Gamma x \qquad (6)$$

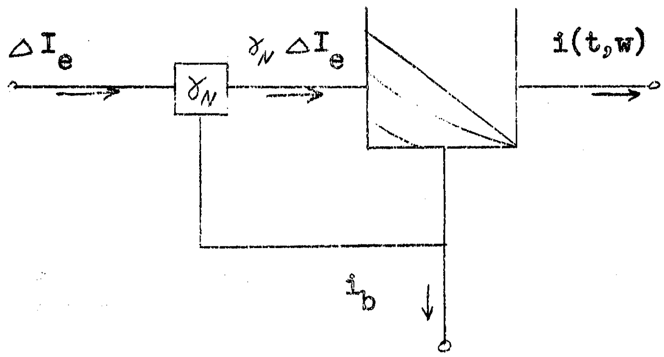
COMMON BASE ACTIVE SOLUTION

To make this transient solution fit physical problems we must add or subtract it from another solution which represents the initial conditions. This may be thought of as two transistors in parallel, one representing the initial conditions and the other the transient solution.



The hole density plot is a useful device for visualizing the transient response.[2] By (2) the current is proportional to the slope of the hole density. Equation (3) shows that in the steady state, any curvature is due to recombination.

Emitter efficiency, γ_N, is the ratio of the hole current at x = 0 to the current in the emitter lead. It is a function of the emitter hole density but will be assumed to be constant in this analysis.



The above diagram represents the effect of the emitter efficiency. Only the fraction γ_N of the actual emitter current reaches the emitter of the ideal transistor. The vertical lines are electron current to make the device obey Kirchoff's current law.

In the active region the collector is reverse biased and the hole density there is zero. Thus the boundary conditions on (6) are

$$\begin{cases} P(s,w) = 0 \\ I(s,0) = \gamma_N \dfrac{\Delta I_e}{s} \end{cases} \tag{7}$$

The constants A and B may be found giving

$$P(s,x) = \frac{\Delta I_e \, \gamma_N}{q D_p \Gamma s} \; \frac{\operatorname{Sinh} \Gamma (w-x)}{\operatorname{Cosh} \Gamma w} \tag{8}$$

The collector current is obtained by applying (2) to (8) and setting x = w.

$$I(s,w) = \frac{\Delta I_e \, \gamma_N}{s} \; \frac{1}{\operatorname{Cosh} \Gamma w} \tag{9}$$

The inversion of this Laplace transform is the solution for the collector current as a function of time. To do this we must expand (9) in a partial fraction expansion which is actually done by finding the residue at each pole. To facilitate the solution two theorems from Laplace transform theory will be used; the shift of poles in the s plane, and the time scale change.[3]

Replacing s by $s - \frac{1}{\tau_p}$ in the s domain multiplies in the time domain by $\exp(-\frac{t}{\tau_p})$. Multiplying s by $\frac{D_p}{w^2}$ in the s domain multiplies t by $\frac{w^2}{D_p}$ in the time domain.

$$i(t,w) = e^{-\frac{t}{\tau_p}} \mathcal{L}_s^{-1} \frac{\Delta I_e \, \gamma_N}{(s - \frac{1}{\tau_p}) \; \operatorname{Cosh}\sqrt{\frac{sw^2}{D_p}}}$$

$$\tilde{i}(T,w) = e^{-\frac{w^2}{L_p^2} T} \mathcal{L}_\lambda^{-1} \frac{\Delta I_e \, \gamma_N}{(\lambda - \frac{w^2}{L_p^2}) \; \operatorname{Cosh}\sqrt{\lambda}} \qquad (10)$$

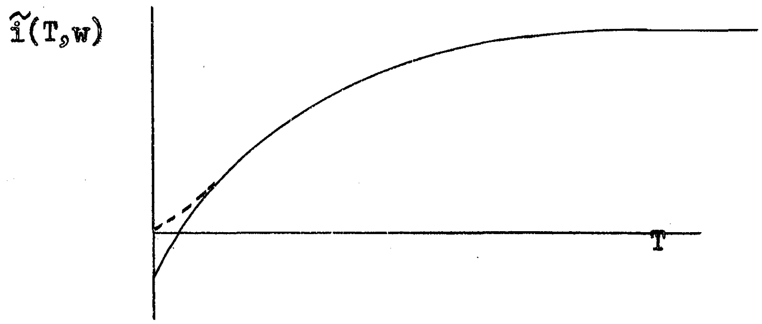
where $\lambda = \frac{w^2}{D_p} s \qquad L_p^2 = D_p \tau_p$

$$T = \frac{D_p}{w^2} t$$

This has poles at $\lambda = \frac{w^2}{L_p^2}$ and $\operatorname{Cosh}\sqrt{\lambda} = \operatorname{Cos}\sqrt{-\lambda} = 0$

or
$$\lambda = -\left(\frac{2n+1}{2}\right)^2 \pi^2 \; ; \; \frac{w^2}{L_p^2} \qquad n = 0, 1, \ldots \qquad (11)$$

Fig. 1(b) is a plot of the poles in the λ plane. To avoid appearing to have a pole in the right half plane, which would lead to a growing exponential, the poles have been shifted back by $s = \frac{1}{\tau_p}$ or $\lambda = \frac{w^2}{L_p^2}$. The residue at any pole is proportional to the product of the reciprocals of vectors drawn to that pole from all others. Because of the proximity of the pole at the origin to its neighbor, these two terms dominate all the rest. Note that the residue at the first pole from the origin is negative because one vector points in the negative direction, and is larger than the residue at the origin because the pole is closer to the outer poles. Neglecting all poles but the two, we might expect this:



Since (5) requires that the function start at zero the effect of the neglected poles is just to alter the response near zero as shown by the dotted line.

The inversion of (10) is accomplished as follows:

$$\mathcal{L}_\lambda \, e^{+\frac{w^2}{L_p^2}T}\,\tilde{i}(T,w) = \frac{K_1}{\lambda - \frac{w^2}{L_p^2}} + \sum_{n=0}^{\infty}\frac{K_{2n}}{\lambda + \left(\frac{2n+1}{2}\right)^2\pi^2} = \frac{\Delta I_e \, \gamma_N}{\left(\lambda - \frac{w^2}{L_p^2}\right)\operatorname{Cosh}\sqrt{\lambda}} \tag{12}$$

Multiplying through by $\lambda - \frac{w^2}{L_p^2}$ and letting $\lambda \rightarrow \frac{w^2}{L_p^2}$ we obtain

$$K_1 = \frac{\Delta I_e \, \gamma_N}{\operatorname{Cosh}\frac{w}{L_p}} = \Delta I_e \, \gamma_N \, \beta_N = \Delta I_e \, \alpha_N \tag{13}$$

where β_N is the transport factor, not to be confused with the common emitter current gain. The N stands for normal as we later consider the inverted current gains. In a similar manner K_{2n} can be obtained. Thus

$$\frac{\tilde{i}(T,w)}{\Delta I_e \, \alpha_N} = 1 - \frac{1}{\beta}\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1) + \frac{4}{\pi^2}\frac{w^2}{L_p^2}\frac{1}{(2n+1)}} \; e^{-\left[\left(\frac{2n+1}{2}\right)^2\pi^2 + \frac{w^2}{L_p^2}\right]T} \tag{14}$$

Let $\beta = 1$ and neglect all terms but two.

$$\frac{\tilde{i}(T,w)}{\Delta I_e \alpha_N} = 1 - \frac{4}{\pi} e^{-\frac{\pi^2}{4} T} \tag{15}$$

In Fig. 2 are plotted the exact expression and the two term approximation for $\beta = 1$.

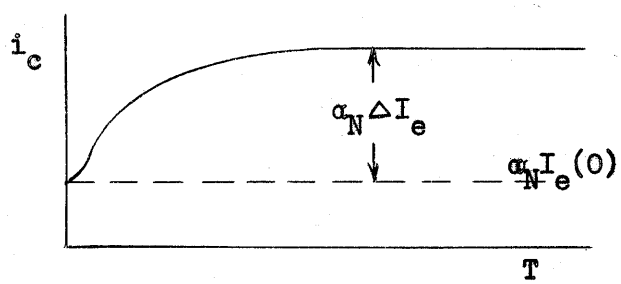
In the same manner the inverse of (8) may be found.

$$\frac{qD_p \tilde{P}(T,x)}{\alpha_N \Delta I_e} = \frac{\operatorname{Sinh} \frac{w}{L_p}(1 - \frac{x}{w})}{\frac{w}{L_p}}$$

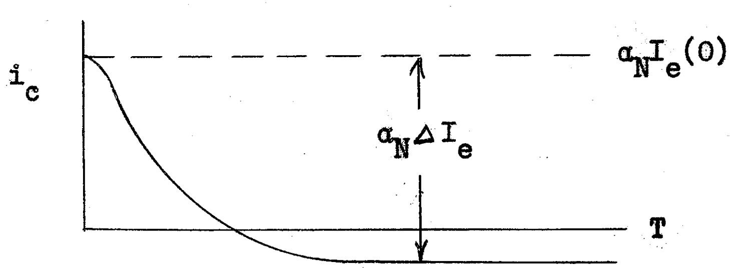
$$- \frac{1}{\beta} \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n \operatorname{Sin} (\frac{2n+1}{2})\pi(1 - \frac{x}{w})}{(2n+1)^2 + \frac{4}{\pi^2} \frac{w^2}{L_p^2}} e^{-\left[(\frac{2n+1}{2})^2 \pi^2 + \frac{w^2}{L_p^2}\right] T} \tag{16}$$

This is plotted in Fig. 3 for $\beta = 1$.

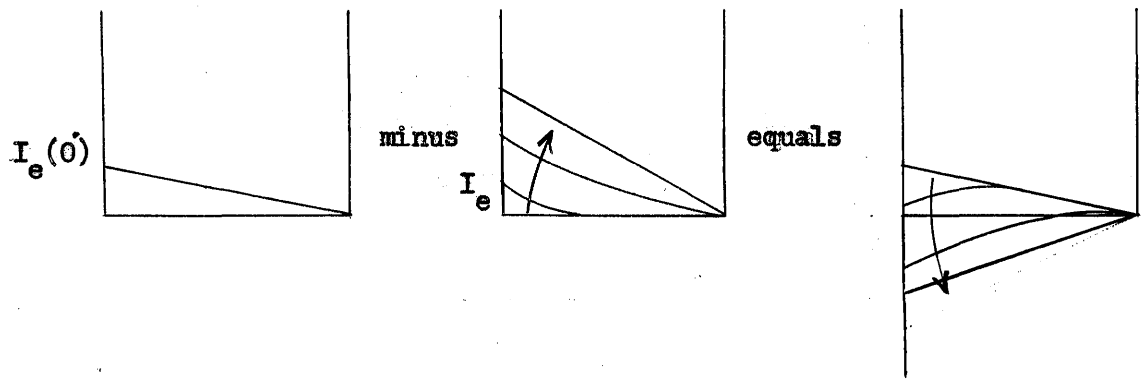
When the change of emitter current, ΔI_e, is a positive step, the transient collector current, $\tilde{i}(T,w)$, is added to $I_e(0)$ times the steady-state current gain α_N.



When ΔI_e is a negative step, $\tilde{i}(T,w)$ is subtracted from $\alpha_N I_e(0)$. If $\alpha_N \Delta I_e$ is larger than $\alpha_N I_e(0)$, as is usually the case during turn-off, the solution is valid only for positive currents as the emitter cannot emit backwards.

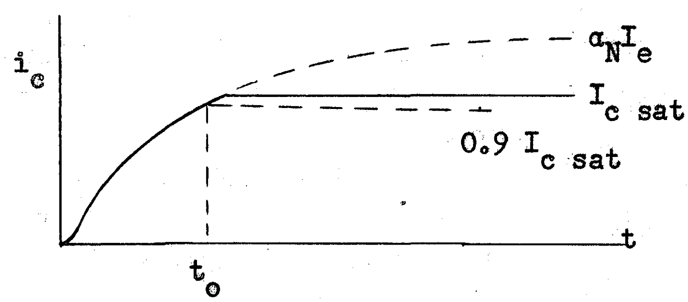


Actually the response near zero is an approximation because negative hole densities are assumed, an impossibility. Below are shown the hole density plots from which the above curve is obtained by using (2) at the collector.



While the slope at the collector is still negative, and collector current still flows, the hole density at the emitter is negative. This difficulty also arises in the derivation of switching times by Ebers and Moll[4]. The actual effect may be similar to this, however. When the emitter hole density reaches zero, an avalanche or punch through effect can take place which tends to move the emitter junction boundary toward the collector.

The rise time, t_o, is defined as the time for the collector current to rise from zero to 0.9 of its final value, $I_{c\ sat}$, which equals the collector supply voltage divided by the collector load resistance. This assumes zero emitter-collector voltage, a good approximation for supply voltages over one volt.



The rise time may be found by using the above figure and (15) the two term approximation.

$$\alpha_N I_e \left(1 - \frac{4}{\pi} e^{-\frac{\pi^2}{4} T}\right) = 0.9\ I_{c\ sat}$$

$$T_o = \frac{D_p}{w^2} t_o = \frac{4}{\pi^2} \ln \frac{4}{\pi} \frac{I_e}{I_e - \frac{0.9\ I_{c\ sat}}{\alpha_N}} \tag{17}$$

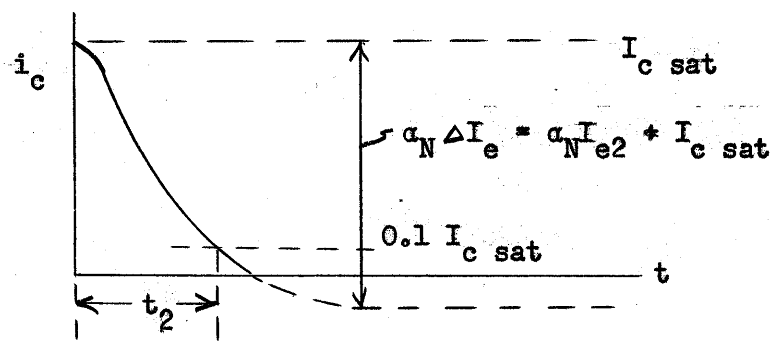
Ebers and Moll[4] obtained

$$T_o = \frac{D_p}{w^2} t = \frac{1}{2} \ln \frac{I_e}{I_e - \frac{0.9\ I_{c\ sat}}{\alpha_N}} \tag{18}$$

$$\text{with} \quad \omega_N = \frac{2\ D_p}{w^2} \tag{19}$$

by using $(1 - e^{-2T})$ instead of $(1 - \frac{4}{\pi} e^{-\frac{\pi^2}{4} T})$. This results from assuming that the hole density plot (Fig. 3) consists of straight lines.[5] These curves are compared in Fig. 2. In (17) the effect of the delay is incorporated in the $\frac{4}{\pi}$ while (18) permits zero rise time for infinite I_e.

For the fall time the following curve is obtained.



Thus

$$I_{c\ sat} - (\alpha_N I_{e2} + I_{c\ sat})\left(1 - \frac{4}{\pi} e^{-\frac{\pi^2}{4} T_2}\right) = 0.1\ I_{c\ sat}$$

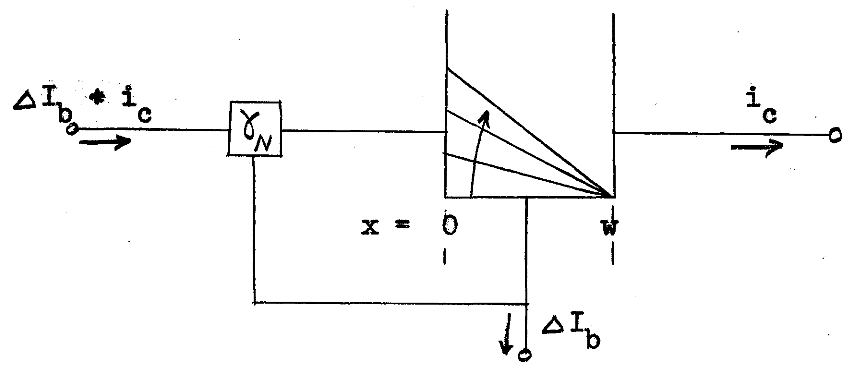
$$T_2 = \frac{D_p}{w^2} t_2 = \frac{4}{\pi^2} \ln \frac{4}{\pi} \frac{I_{c\ sat} + \alpha_N I_{e2}}{0.1\ I_{c\ sat} + \alpha_N I_{e2}} \tag{20}$$

as compared with Ebers and Moll's relation,

$$T_2 = \frac{D_p}{w^2} t_2 = \frac{1}{2} \ln \frac{I_{c\ sat} + \alpha_N I_{e2}}{0.1\, I_{c\ sat} + \alpha_N I_{e2}} \tag{21}$$

COMMON EMITTER ACTIVE SOLUTION

The common emitter configuration is the usual one used because of its high current gain and phase inversion property. A step of base current, $\triangle I_b$ is applied.



The boundary consitions on (6) are now

$$\begin{cases} P(s,w) = 0 \\ I(s,0) = \gamma_N\left(\frac{\triangle I_b}{s} + I(s,w)\right) \end{cases} \tag{22}$$

with solution

$$P(s,x) = \frac{\triangle I_b \gamma_N}{q D_p \Gamma\, s} \frac{\sinh \Gamma (w-x)}{(\cosh \Gamma w - \gamma_N)} \tag{23}$$

and

$$I(s,w) = \frac{\triangle I_b \gamma_N}{s} \frac{1}{(\cosh \Gamma w - \gamma_N)} \tag{24}$$

After normalization and a shift of poles in the plane, (24) becomes

$$\tilde{i}(T,w) = e^{-\frac{w^2}{L_p^2} T} \mathcal{L}_\lambda^{-1} \frac{\triangle I_b \gamma_N}{(\lambda - \frac{w^2}{L_p^2})(\cosh\sqrt{\lambda} - \gamma_N)} \tag{25}$$

This has poles at $\lambda = \frac{w^2}{L_p^2}$ and at $\operatorname{Cosh}\sqrt{\lambda} = \operatorname{Cos}\sqrt{-\lambda} = \gamma_N$

$$\sqrt{-\lambda} = n2\pi \pm \operatorname{Cos}^{-1}\gamma_N$$

Thus

$$\lambda = \frac{w^2}{L_p^2}, \quad -(n2\pi \pm \operatorname{Cos}^{-1}\gamma_N)^2 \qquad n = 0,1,2,\cdots \tag{26}$$

Fig. 1(b) is a plot of the poles in the λ plane. The poles have been shifted back by w^2/L_p^2 as before. Because of the closeness of the first two poles compared with outer ones, they may be neglected to a much better approximation than in the common-base solution. The response is then

$$\tilde{i}(T,w) = K_1(1 - e^{\lambda_1 T}) \tag{27}$$

where K_1 is the residue at the origin and λ_1 is the location of the first pole from the origin.

$$\lambda_1 = -(\operatorname{Cos}^{-1}\gamma_N)^2 - \frac{w^2}{L_p^2} \tag{28}$$

But by (13) and the power series for Cosh x, we have

$$\operatorname{Cosh}\frac{w}{L_p} = \frac{1}{\beta} \cong 1 + \frac{w^2}{2L_p^2}$$

$$\frac{w^2}{L_p^2} \cong 2(\frac{1}{\beta} - 1) \tag{29}$$

Similarly

$$\operatorname{Cos}\sqrt{-\lambda} = \gamma_N \cong 1 - \frac{(-\lambda)}{2}$$

$$-(\operatorname{Cos}^{-1}\gamma_N)^2 \cong -2(1 - \gamma_N) \tag{30}$$

So (28) becomes

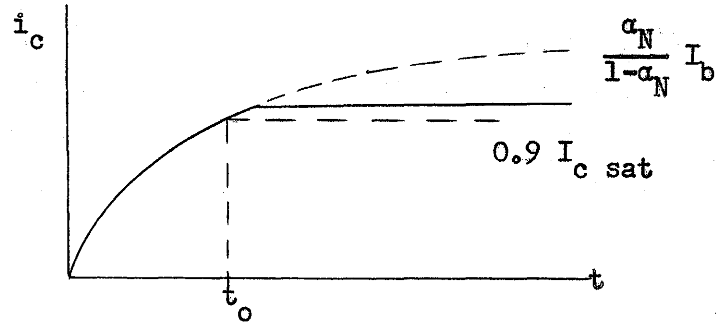
$$\lambda_1 \cong -2(1 - \gamma_N + \frac{1}{\beta} - 1) = -\frac{2}{\beta}(1 - \alpha_N) \tag{31}$$

and after neglecting the factor $\frac{1}{\beta}$ and using (19), the result of Ebers and Moll is obtained

$$\lambda_1 = -2(1-\alpha_N) = -\frac{w^2}{D_p}\omega(1-\alpha_N) \tag{32}$$

The validity of (27) may be checked by taking the inverse of (24) with $\gamma_N = 1$. The residues at the first two poles lead to a delay (T = .1 Fig. 2) of 1/12 which is very small with respect to the normalized time constant of the exponential of $\frac{1}{2(1-\alpha_N)}$.

The normalized rise and fall times may be obtained in a manner similar to the common base derivation using (27) instead of (15). Thus



$$T_o = \frac{D_p}{w^2} t_o = \frac{1}{2(1-\alpha_N)} \ln \frac{I_b}{I_b - 0.9\frac{1-\alpha_N}{\alpha_N} I_{c\ sat}} \tag{33}$$

and similarly

$$T_2 = \frac{D_p}{w^2} t_2 = \frac{1}{2(1-\alpha_N)} \ln \frac{I_{c\ sat} + \frac{\alpha_N}{1-\alpha_N} I_{b2}}{0.1\ I_{c\ sat} + \frac{\alpha_N}{1-\alpha_N} I_{b2}} \tag{34}$$

COMMON COLLECTOR ACTIVE SOLUTION

The transient emitter current is obtained by adding I_b to the transient collector current obtained in the common emitter solution.

$$\tilde{i}(T,w) = \Delta I_b \frac{\alpha_N}{1-\alpha_N}(1 - e^{-2(1-\alpha_N)T}) + \Delta I_b = \Delta I_b \frac{1}{1-\alpha_N}(1-\alpha_N e^{-2(1-\alpha_N)T}) \tag{35}$$

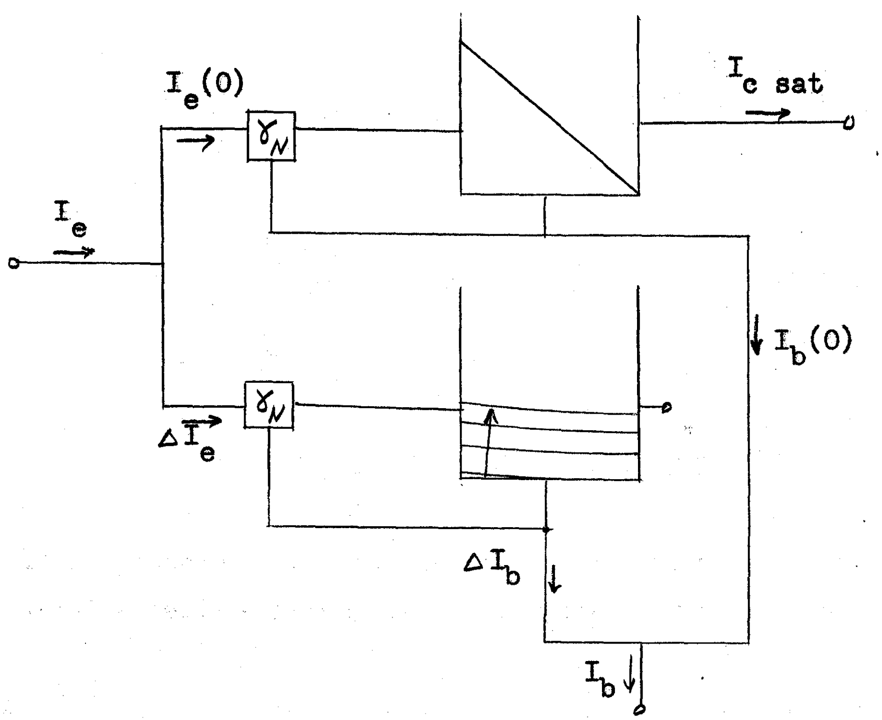
The rise and fall times may now be obtained in the same manner as before.

$$T_o = \frac{D_p}{w^2} t_o = \frac{1}{2(1-\alpha_N)} \ln \frac{\alpha_N I_b}{I_b - 0.9(1-\alpha_N) I_{e\ sat}} \tag{36}$$

$$T_2 = \frac{D_p}{w^2} t_2 = \frac{1}{2(1-\alpha_N)} \ln \frac{\alpha_N (I_{e\ sat} + \frac{1}{1-\alpha_N} I_{b2})}{0.1\, I_{e\ sat} + \frac{1}{1-\alpha_N} I_{b2}} \tag{37}$$

SATURATION SOLUTION

In the saturation region the collector current is limited by its saturation value, $I_{c\ sat}$. The collector junction becomes forward biased and the hole density at the collector is no longer zero. The steady-state and transient solutions may be represented as two parallel transistors as before.



It is seen from the above figure that $\Delta I_e = \Delta I_b$, therefore the solution is valid for all three circuit configurations. The boundary conditions on (6) are

$$\begin{cases} I(s,w) = 0 \\ I(s,0) = \dfrac{\Delta I_{b,e}\,\gamma_N}{s} \end{cases} \tag{38}$$

with solution

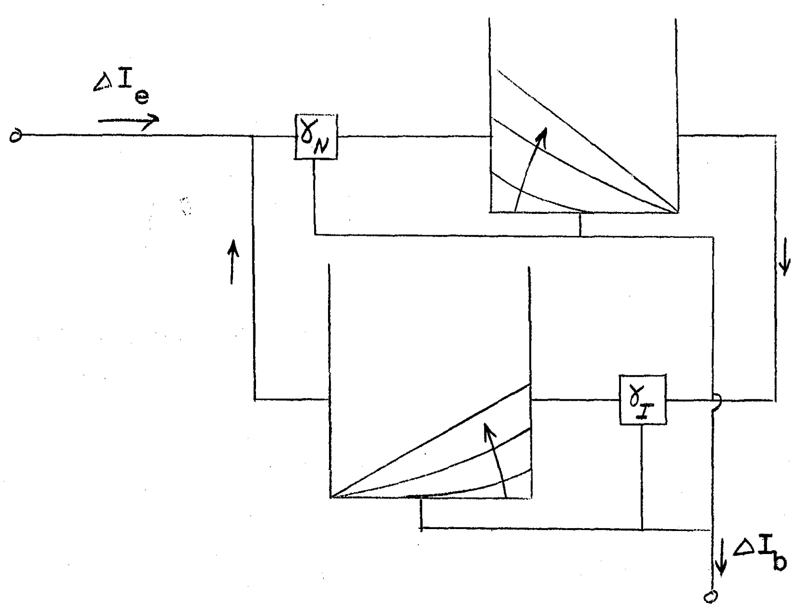
$$P(s,x) = \frac{\Delta I_{b,e}\,\gamma_N}{q\,D_p\,\Gamma\,s}\;\frac{\operatorname{Cosh}\Gamma(w-x)}{\operatorname{Sinh}\Gamma w} \tag{39}$$

and

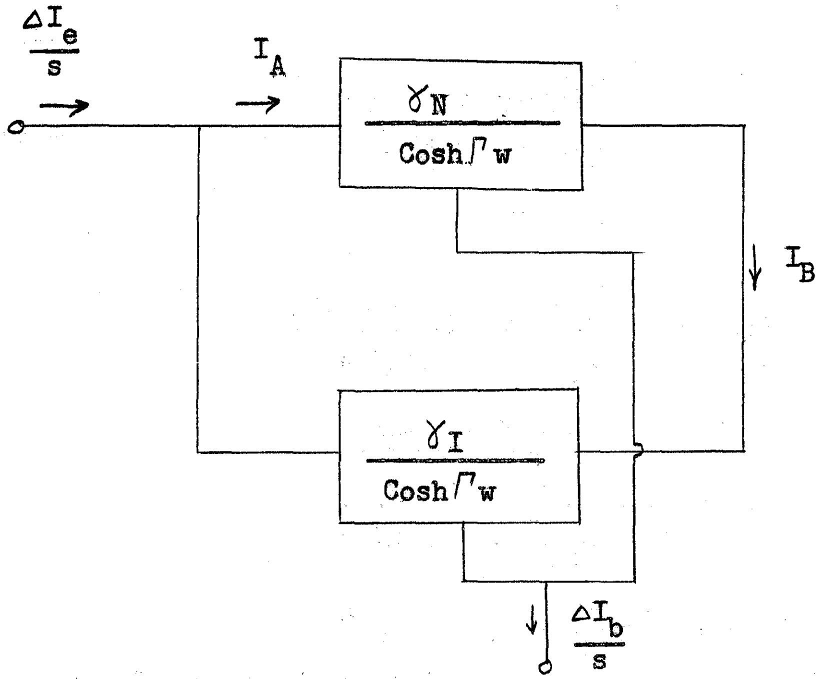
$$P(s,w) = \frac{\Delta I_{b,e}\,\gamma_N}{q\,D_p\,\Gamma\,s}\;\frac{1}{\operatorname{Sinh}\Gamma w} \tag{40}$$

as has been previously obtained by Konkle.[6]

An important effect has been neglected, however. The collector is forward biased and is emitting holes with efficiency γ_I. To take account of this, the transient solution is broken up into two active solutions in the manner of Ebers and Moll.



The active region hole density plots may be replaced by the transfer functions derived in the common base analysis.



$$I_A(s) = \frac{\Delta I_e}{s} + \frac{\gamma_N \gamma_I}{\text{Cosh}^2 \Gamma w} \; I_A(s)$$

$$I_B(s) = \frac{\gamma_N}{\text{Cosh}\, \Gamma w} \; I_A(s) = \frac{\Delta I_e \, \gamma_N \; \text{Cosh}\, \Gamma w}{s \; (\text{Cosh}^2 \Gamma w - \gamma_N \gamma_I)} \tag{41}$$

The hole density at the collector, P_c, may be found using (8).

$$P_c(s) = \frac{I_B(s)\, \gamma_I \; \text{Sinh}\, \Gamma w}{q \; D_p \Gamma \; \text{Cosh}\, \Gamma w} = \frac{\Delta I_e \; \gamma_N \gamma_I \; \text{Sinh}\, \Gamma w}{q \; D_p \Gamma \, s \; (\text{Cosh}^2 \Gamma w - \gamma_N \gamma_I)} \tag{42}$$

This reduces to (40) for $\gamma_N = \gamma_I = 1$. After normalization and a shift, the poles are found to be at

$$\lambda = \frac{w^2}{L_p^2} \qquad \text{and} \quad \text{Cosh}\sqrt{\lambda} = \text{Cos}\sqrt{-\lambda} = \pm\sqrt{\gamma_N \gamma_I} \, .$$

$$\text{or} \sqrt{-\lambda} = (n\pi \pm \text{Cos}^{-1}\sqrt{\gamma_N \gamma_I})$$

thus after a shift back, the poles are located at

$$\lambda = 0, \quad -(n\pi \pm \text{Cos}^{-1}\sqrt{\gamma_N \gamma_I})^2 - \frac{w^2}{L_p^2} \qquad n = 0,1,2,\cdots \tag{43}$$

Similarly (42) has zeros at

$$\lambda = -(n\pi)^2 - \frac{w^2}{L_p^2} \tag{44}$$

The pole-zero plot of (42) is shown in Fig. 1(c).

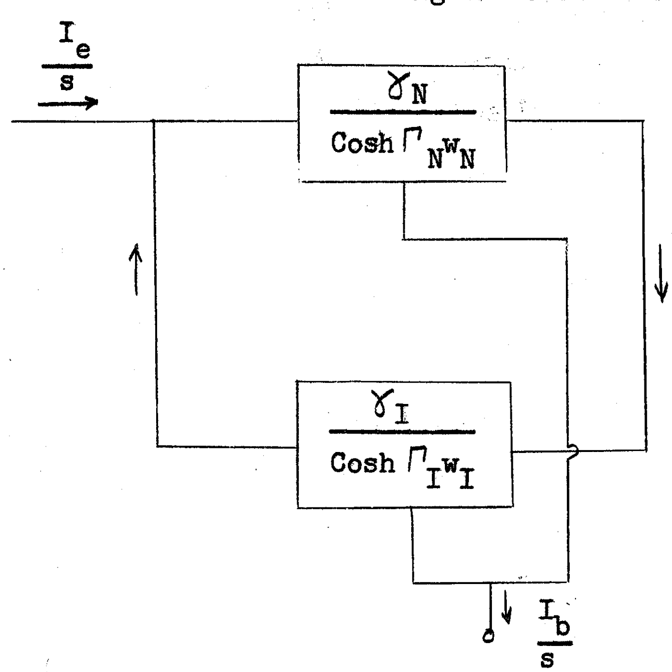
The analysis now parallels that involving (27) through (32) with γ_N replaced by $\sqrt{\gamma_N \gamma_I}$. This leads to

$$\lambda_1 = -\frac{2}{\beta}\left(1-\alpha_N\sqrt{\frac{\gamma_I}{\gamma_N}}\right) = -\frac{w^2}{D_p}\,\omega\,\left(1-\alpha_N\sqrt{\frac{\gamma_I}{\gamma_N}}\right) \tag{45}$$

This differs from Ebers and Moll's expression

$$\lambda_1 = -\frac{w^2}{D_p}\left(\frac{\omega_N\omega_I}{\omega_N + \omega_I}\right)(1 - \alpha_N\alpha_I) \tag{46}$$

In order to take into account deviations from the one-dimensional planar transistor, they considered the inverted transistor to have a different than the normal one. This can also be done by considering two one-dimensional transistors having different base widths.



Thus (42) becomes

$$P_c(s) = \frac{\Delta I_e\ \gamma_N\gamma_I\ \text{Sinh}\ \Gamma_I w_I}{q\ D_p\ \Gamma_I\ s\ (\text{Cosh}\ \Gamma_N w_N\ \text{Cosh}\ \Gamma_I w_I - \gamma_N\gamma_I)} \tag{47}$$

It does not help here to shift the poles by $\frac{w^2}{L_p^2}$ because it is not equal in the inverted and normal directions. Equation (47) has poles at

$$\cosh\sqrt{\left(s_1 + \frac{1}{\tau_N}\right)\left(\frac{w_N^2}{D_p}\right)} \quad \cosh\sqrt{\left(s_1 + \frac{1}{\tau_I}\right)\left(\frac{w_I^2}{D_p}\right)} = \gamma_N\gamma_I \qquad (48)$$

Using (19), (29), and two terms of the power series for Cosh x, we obtain

$$s_1 = -\left(\frac{\omega_N\omega_I}{\omega_N + \omega_I}\right)\left[1 - \left(1 - \frac{1-\beta_N}{\beta_N\gamma_N\gamma_I} - \frac{1-\beta_I}{\beta_I\gamma_N\gamma_I}\right)\gamma_N\gamma_I\right]$$

$$\cong -\left(\frac{\omega_N\omega_I}{\omega_N + \omega_I}\right)\left[1 - (1-[1-\beta_N])(1-[1-\beta_I])\,\gamma_N\gamma_I\right]$$

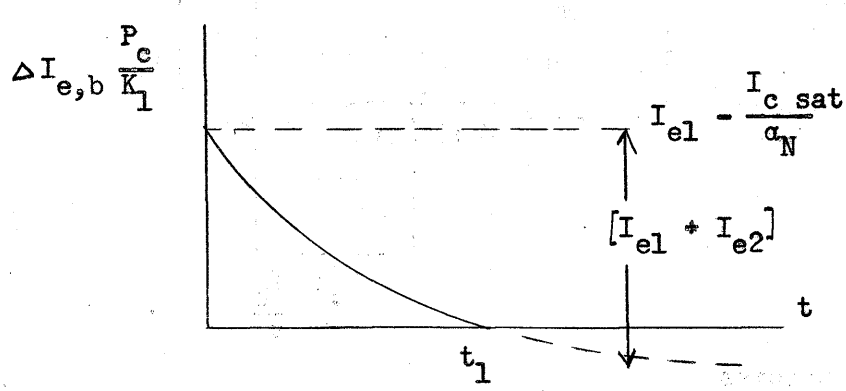
$$\cong -\left(\frac{\omega_N\omega_I}{\omega_N + \omega_I}\right)(1-\alpha_N\alpha_I) \qquad (49)$$

Although it does not enter into the expression for storage time, the value of collector hole density finally reached with input ΔI_e is obtained by finding the residue at the origin of (47).

$$P_c = K_1(1 - e^{s_1 t}) \qquad (50)$$

$$K_1 = \frac{\Delta I_{e,b}\, w}{q\, D_p}\;\frac{\alpha_N\alpha_I}{1 - \alpha_N\alpha_I} \qquad (51)$$

The storage time, t_1, is obtained in the same manner as the fall times.



Common Base

$$t_1 = \frac{\omega_N + \omega_I}{\omega_N\omega_I(1-\alpha_N\alpha_I)} \ln \frac{I_{e2} + I_{e1}}{I_{e2} + \frac{I_{c\ sat}}{\alpha_N}} \quad \text{(Common Base)} \qquad (52)$$

$$t_1 = \frac{\omega_N + \omega_I}{\omega_N\omega_I(1-\alpha_N\alpha_I)} \ln \frac{I_{b2} + I_{b1}}{I_{b2} + I_{c\ sat}(\frac{1-\alpha_N}{\alpha_N})} \quad \text{(Common Emitter)} \qquad (53)$$

$$t_1 = \frac{\omega_N + \omega_I}{\omega_N\omega_I(1-\alpha_N\alpha_I)} \ln \frac{I_{b2} + I_{b1}}{I_{b2} + I_{e\ sat}(1-\alpha_N)} \quad \text{(Common Collector)} \qquad (54)$$

If (45) were used instead of (47), the time constant term in the above equations would be

$$\frac{1}{\omega(1-\alpha_N\sqrt{\frac{\gamma_I}{\gamma_N}})} \qquad (55)$$

By making γ_I as small as possible, storage time is reduced without affecting the response in the active region. After assuming γ_N equals one, the factor

$$\frac{1}{1-\alpha_N\sqrt{\gamma_I}}$$

is plotted vs γ_I in Fig. 4 with α_N as a parameter. The effect of γ_I on the storage time is quite apparent.

SUMMARY

Previous works on junction transistor transient response have treated it as an extension of small signal theory. The hyperbolic expression for β

$$\frac{1}{\text{Cosh}\ \frac{W^2}{D_p} s + \frac{W^2}{L_p^2}}$$

is approximated by various polynomials in $j\omega$ in a sinusoidal analysis. This expression is then used to obtain the transient response.

In this paper the hyperbolic functions are solved directly by Laplace transforms. This leads to a modification of Ebers and Moll's switching time equations for common base operation. The hyperbolic

expressions with common emitter operation and in the storage region are also solved directly. The results agree quite well with Ebers and Moll. Other forms of the results include explicitly the emitter efficiencies.

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Attached Drawings: Fig. 1- A-48864-G
Fig. 2- A-48865-G
Fig. 3- A-48866-G
Fig. 4- A-48867-G

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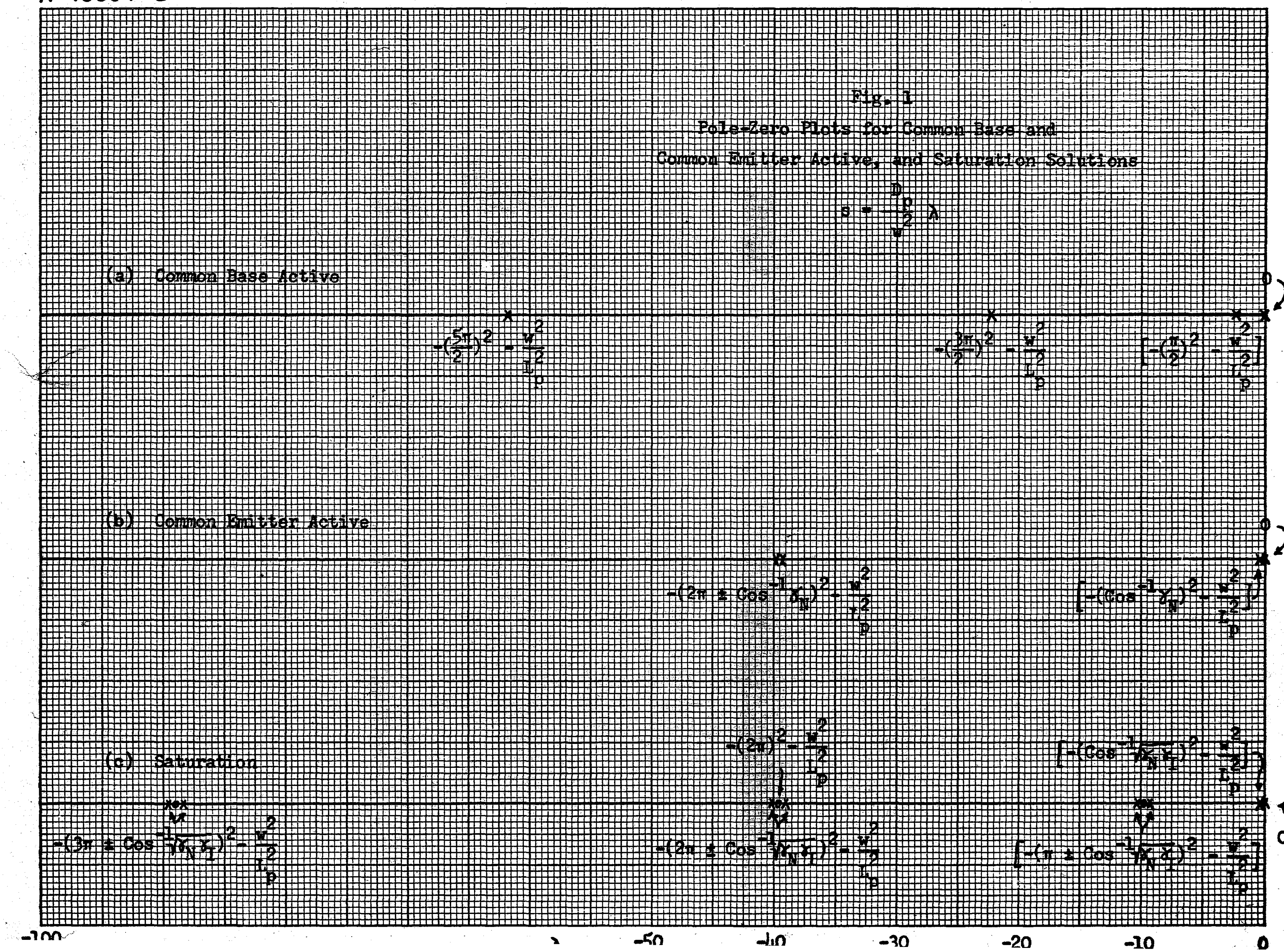
Fig. 1
Pole-Zero Plots for Common Base and
Common Emitter Active, and Saturation Solutions
$s = \frac{D_p}{W^2} \lambda$
(a) Common Base Active
$-(\frac{5\pi}{2})^2 - \frac{W^2}{L_p^2}$
$-(\frac{3\pi}{2})^2 - \frac{W^2}{L_p^2}$
$[-(\frac{\pi}{2})^2 - \frac{W^2}{L_p^2}]$
(b) Common Emitter Active
$-(2\pi \pm \cos^{-1}\gamma_N)^2 - \frac{W^2}{L_p^2}$
$[-(\cos^{-1}\gamma_N)^2 - \frac{W^2}{L_p^2}]$
(c) Saturation
$-(2\pi)^2 - \frac{W^2}{L_p^2}$
$[-(\cos^{-1}\sqrt{\gamma_N \gamma_I})^2 - \frac{W^2}{L_p^2}]$
$-(3\pi \pm \cos^{-1}\sqrt{\gamma_N \gamma_I})^2 - \frac{W^2}{L_p^2}$
$-(2\pi \pm \cos^{-1}\sqrt{\gamma_N \gamma_I})^2 - \frac{W^2}{L_p^2}$
$[-(\pi \pm \cos^{-1}\sqrt{\gamma_N \gamma_I})^2 - \frac{W^2}{L_p^2}]$
0
-100
-50
-40
-30
-20
-10
0

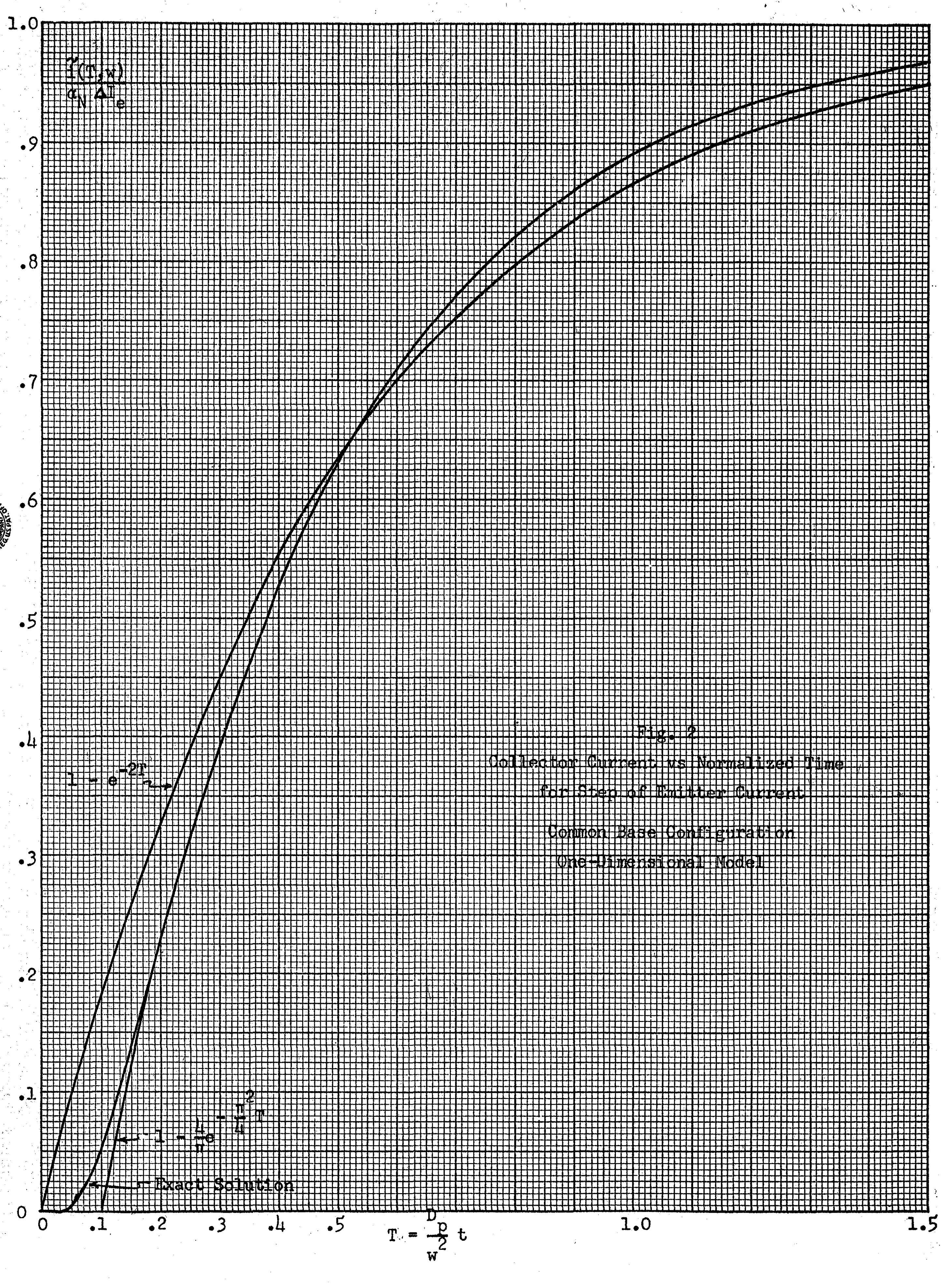

Fig. 2
Collector Current vs Normalized Time
for Step of Emitter Current
Common Base Configuration
One-Dimensional Model

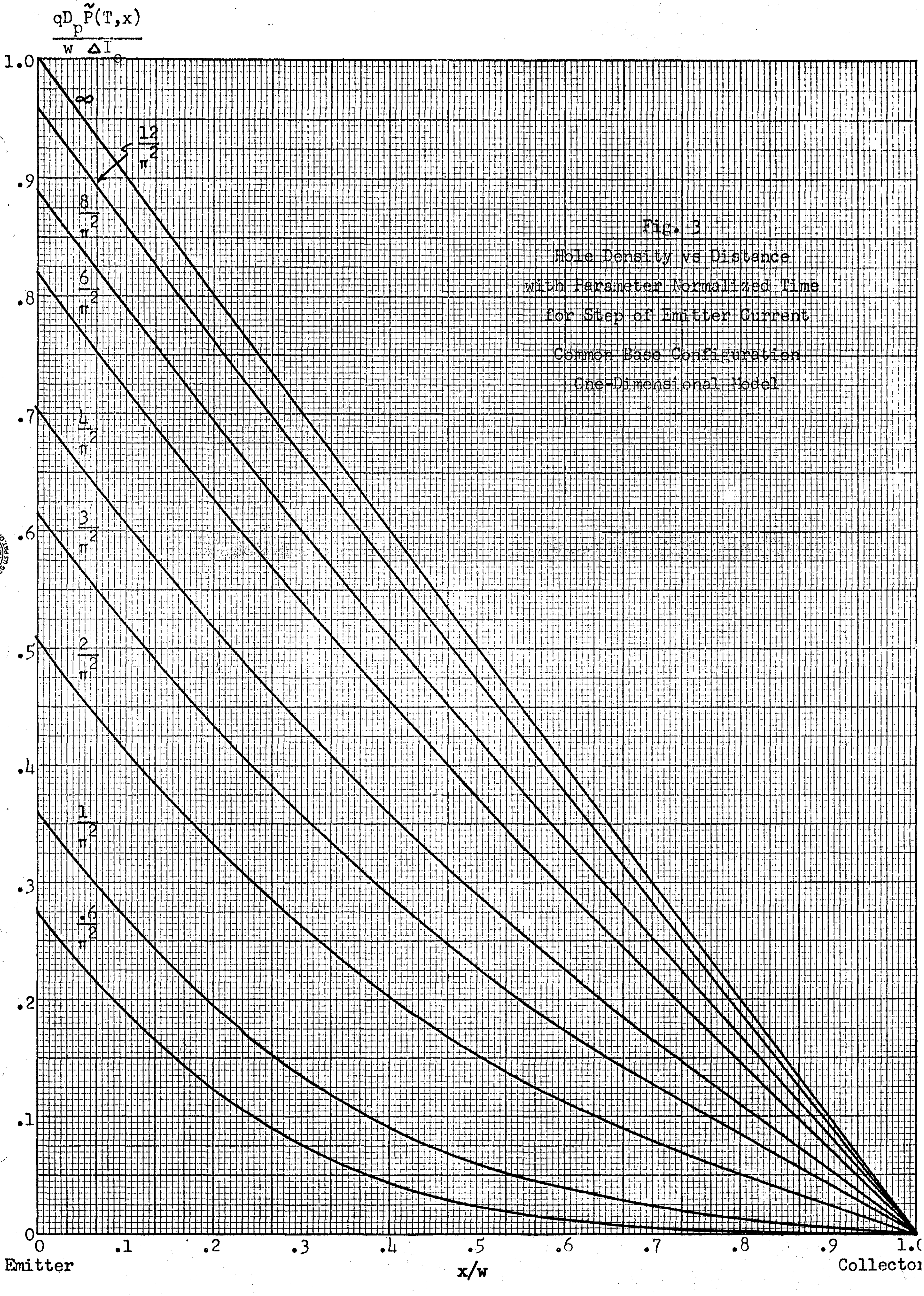

Fig. 3
Hole Density vs Distance
with Parameter Normalized Time
for Step of Emitter Current

Common Base Configuration
One-Dimensional Model

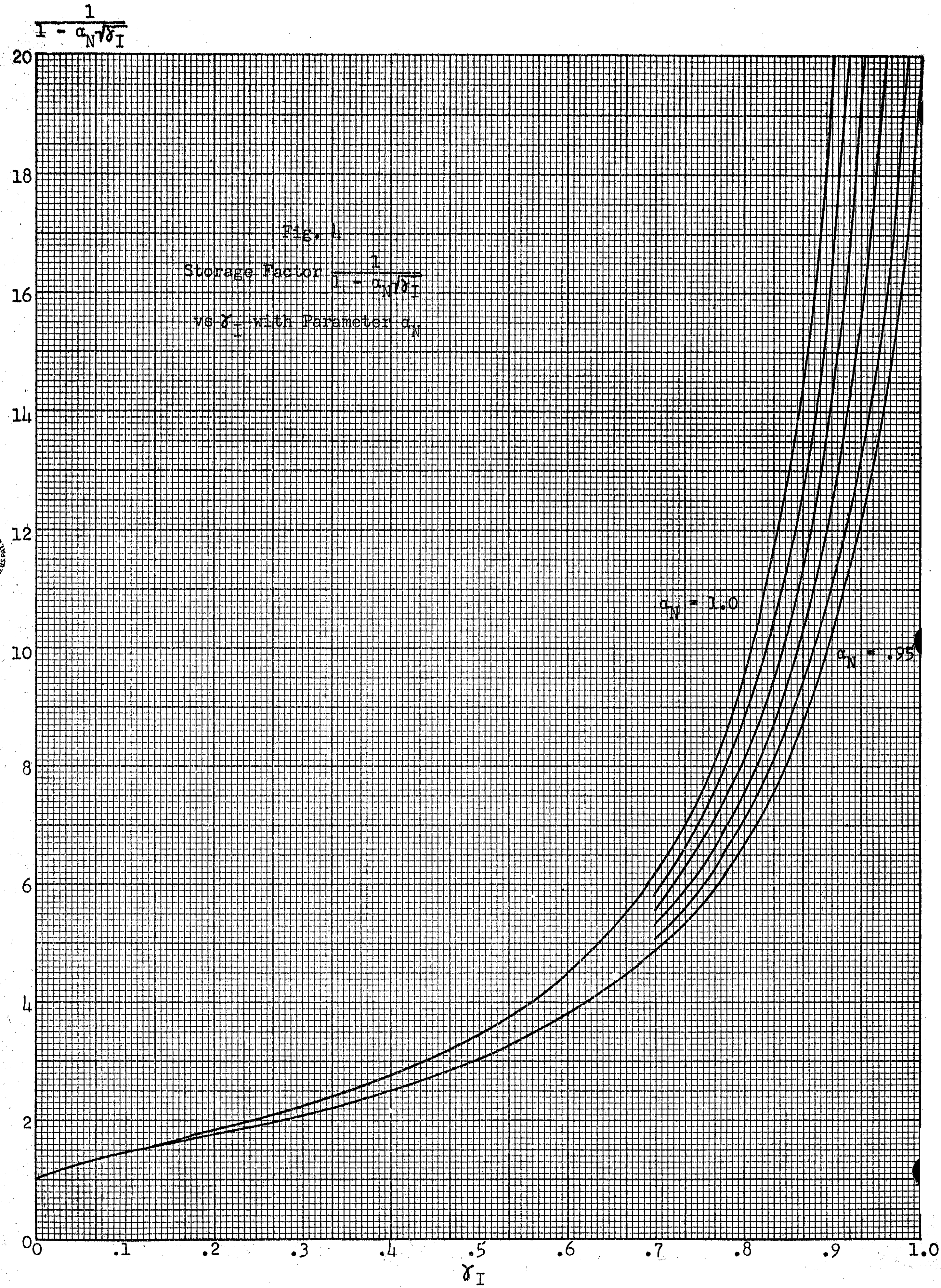

Fig. 4

Storage Factor $\frac{1}{1 - \alpha_N\sqrt{\gamma_I}}$

vs γ_I with Parameter α_N

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