Division 6 - Lincoln Laboratory
Massachusetts Institute of Technology
Lexington 73, Massachusetts

SUBJECT: TRANSISTOR CIRCUITS COURSE
Number 4. Transistor Amplifiers

To: Distribution List

From: Donald J. Eckl

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Approved: William N. Papian

Abstract: Three usable amplifier configurations exist for the junction transistor. The grounded-base amplifier has the highest frequency response. The grounded-emitter has the largest power gain. The grounded-collector or emitter follower serves as an impedance transformer much in the way of a vacuum tube cathode follower. These circuits have many important differences from their vacuum tube counterparts. An important one is the more marked effect of source and load impedances on their performance. Input and output circuits are not isolated by the transistor.

1.0 Grounded-base amplifier

Suppose we consider the grounded-base circuit shown in Fig. 1.

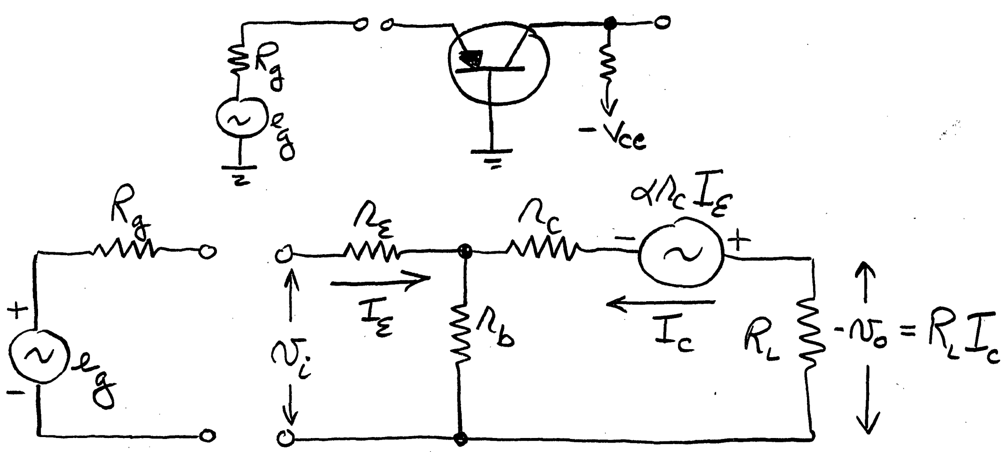

FIG. 1 - GROUNDED BASE TRANSISTOR AMPLIFIER

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The circuit equations are:

$$v_i = I_e(r_e + r_b) + I_c r_b.$$

$$0 = I_e(\alpha r_c + r_b) + I_c(r_c + r_b + R_L).$$

The determinant is

$$\Delta = \begin{vmatrix} (r_e + r_b) & r_b \\ (\alpha r_c + r_b) & (r_c + r_b + R_L) \end{vmatrix}$$

or $$\Delta = r_b\left[r_e + r_c(1-\alpha) + R_L\right] + r_e(r_c + R_L). \qquad (1)$$

$$\Delta \cdot I_e = \begin{vmatrix} v_i & r_b \\ 0 & r_c + r_b + R_L \end{vmatrix}$$

$$I_e = \frac{v_i}{\Delta}(r_c + r_b + R_L) \qquad (2)$$

$$\Delta \cdot I_c = \begin{vmatrix} r_e + r_b & v_i \\ \alpha r_c + r_b & 0 \end{vmatrix}$$

$$I_c = -\frac{v_i}{\Delta}(\alpha r_c + r_b) \qquad (3)$$

The voltage gain is given by:

$$G_v = \frac{-I_c R_L}{v_i} = \frac{(\alpha r_c + r_b)R_L}{\Delta}.$$

There is no phase inversion in the grounded-base circuit. The current gain is given by:

$$A = \frac{I_c}{I_e} = \frac{\alpha r_c + r_b}{r_c + r_b + R_L}.$$

Note that for a short-circuited output ($R_L = 0$) this is simply α.

The input resistance is $R_i = v_i/I_e$.

$$\therefore \quad R_i = \frac{\Delta}{r_b + r_c + R_L}.$$

or $R_i = r_e + r_b\left[\frac{r_c(1-\alpha) + R_L}{r_c + r_b + R_L}\right]$.

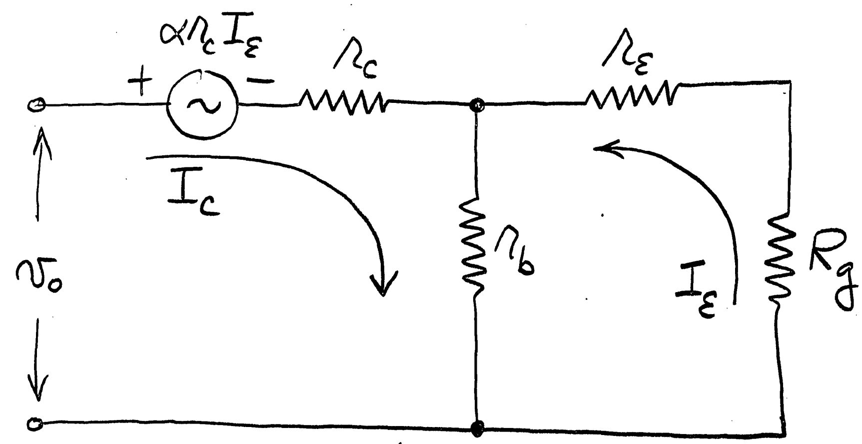
Note that the input resistance depends on the collector load R_L.

The power gain of the circuit is $I_c^2R_L/I_e^2R_i$.

$$P.G. = A^2\left(\frac{R_L}{R_i}\right).$$

This expression shows quite clearly how it is possible to have a power gain in the grounded-base circuit even though the current gain is less than unity. The input impedance is of the order of $r_e + r_b \approx 300\,\Omega$ while the output load R_L may be several thousand ohms. It is the higher impedance level of the output circuit which provides the power gain.

To get the output resistance of the grounded-base stage we must apply a voltage to the output terminals and calculate I_c.



$$\begin{cases} 0 = (R_g + r_e + r_b)I_e + r_bI_c. \\ v_o = (\alpha r_c + r_b)I_e + (r_b + r_c)\,I_c. \end{cases}$$

$$\Delta = r_c(r_e + r_c + R_g) + r_b(R_g + r_e - \alpha r_c).$$

$$\Delta \cdot I_c = \begin{vmatrix} (R_g + r_e + r_b) & 0 \\ (\alpha r_c + r_b) & v_o \end{vmatrix}.$$

The <u>output resistance</u> is v_o/I_c.

$$\therefore R_o = r_c - r_b\left(\frac{\alpha r_c - r_e - R_g}{r_e + r_b + R_g}\right).$$

The output resistance depends on the generator resistance R_g.

If we now make some assumptions about the relative sizes of the quantities in the above expressions, we can obtain simplified versions. Assume:

$$r_c(1-\alpha) >> R_L >> r_e, r_b.$$

Then,

$$\Delta = r_c\left[r_e + r_b(1-\alpha)\right].$$

$$G_v = \frac{\alpha R_L}{r_e + r_b(1-\alpha)}.$$

$$A = \alpha$$

$$R_i = r_e + r_b(1-\alpha).$$

$$R_o = r_c\left\{\frac{r_e + r_b(1-\alpha) + R_g}{r_e + r_b + R_g}\right\} \approx r_c.$$

$$P.G. = \frac{\alpha^2 R_L}{r_e + r_b(1-\alpha)}$$

2.0 <u>Grounded-emitter Amplifier</u>

The grounded-emitter circuit is shown in Fig. 2.

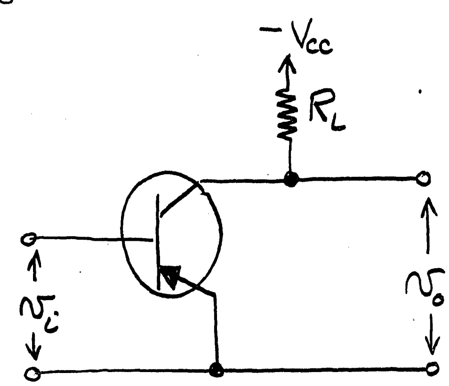


FIG. 2a

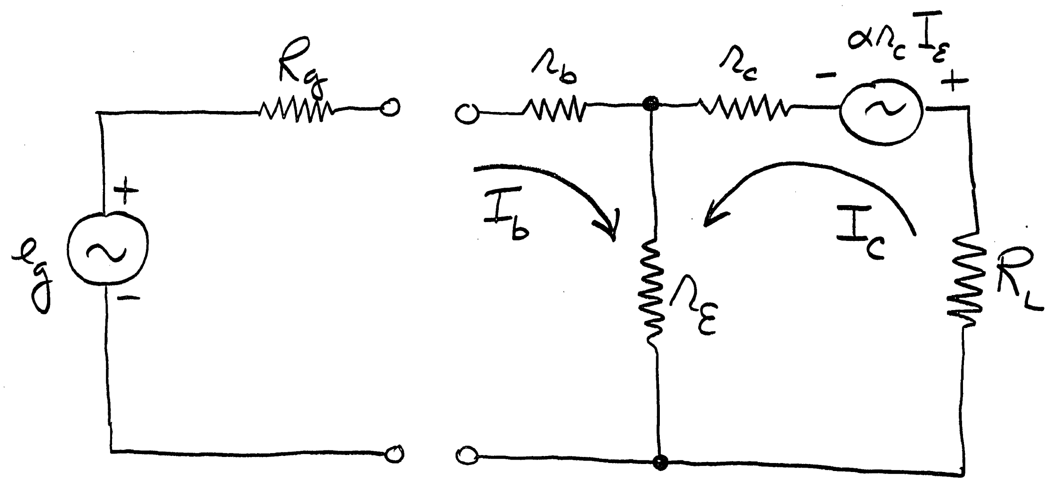


FIG. 2b - GROUNDED- EMITTER AMPLIFIER

$$e_g = (R_g + r_b + r_e)\, I_b + r_e I_c.$$

$$0 = \alpha r_c(-I_b - I_c) + r_e I_b + (R_L + r_c + r_e)\, I_c\, .$$

$$0 = (r_e - \alpha r_c)\, I_b + \left\{R_L + r_e + r_c(1-\alpha)\right\} I_c.$$

$$\Delta = r_b\left\{r_e + R_L + r_c\ (1-\alpha)\right\} + r_e\ (r_c + R_L).$$

This is, of course, the same as for the grounded-base circuit.

$$I_b = \frac{v_i}{\Delta}\left\{r_e + R_L + r_c(1-\alpha)\right\}.$$

$$I_c = \frac{v_i}{\Delta}\,(\alpha r_c - r_e).$$

The voltage gain for the grounded-emitter circuit is

$$G_v = \frac{-I_c R_L}{v_i} = \frac{-(\alpha r_c - r_e)R_L}{\Delta}\, .$$

There is a phase inversion in the output signal. The current gain is given by:

$$A = \frac{I_c}{I_e} = \frac{\alpha r_c - r_e}{r_c(1-\alpha) + r_e + R_L}\, .$$

The input resistance of the grounded emitter circuit is

$$R_i = r_b + r_e \left\{ \frac{r_c + R_L}{r_c(1-\alpha) + r_e + R_L} \right\}.$$

The power gain is

$$P.G. = A^2 \left(\frac{R_L}{R_i} \right).$$

In this case $A > 1$ so the power gain is larger than for the grounded base circuit.

If the output resistance is calculated as before the result is

$$R_o = r_c(1-\alpha) + r_e \left\{ \frac{R_g + r_b + \alpha r_c}{R_g + r_b + r_e} \right\}.$$

If we assume that

$$r_c(1-\alpha) >> R_L >> r_e, r_b$$

we obtain the following approximations:

$$G_v = \frac{-\alpha R_L}{r_e + r_b(1-\alpha)}$$

$$A = \frac{\alpha}{1-\alpha}$$

$$R_i = r_b + \frac{r_e}{1-\alpha}.$$

$$R_o = r_c(1-\alpha) + r_e \left\{ \frac{\alpha r_c + R_g}{r_e + r_b + R_g} \right\}$$

$$P.G. = \frac{1}{(1-\alpha)} \cdot \frac{\alpha^2 R_L}{\{r_e + r_b(1-\alpha)\}}$$

3.0 Grounded-Collector Amplifier

The grounded-collector amplifier is shown in Figure 3. This is the transistor equivalent of the cathode-follower and is frequently referred to as an emitter-follower.

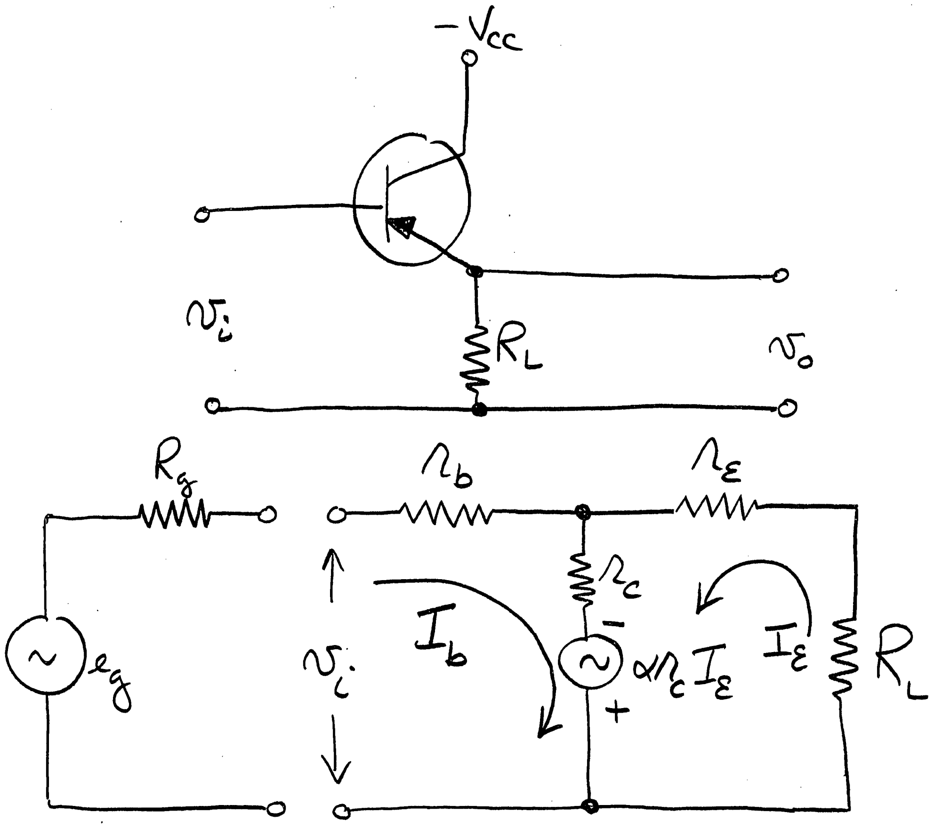


FIG. 3 - GROUNDED - COLLECTOR AMPLIFIER

The loop equations are:

$$v_i = (r_b + r_c)\ I_b + r_c(1-\alpha)I_e$$

$$0 = r_c I_b + \left\{ R_L + r_e + r_c(1-\alpha) \right\} I_e .$$

The input or base current is:

$$I_b = \frac{v_i}{\Delta} \left\{ r_c(1-\alpha) + r_e + R_L \right\}$$

The output current is:

$$I_e = \frac{v_i}{\Delta}\, r_c .$$

The voltage gain for the emitter-follower is

$$G_v = \frac{I_e R_L}{v_i} = \frac{r_c}{\Delta} R_L$$

The current gain is normally greater than unity.

$$A = \frac{I_c}{I_e} = \frac{r_c}{r_c(1-\alpha) + r_e + R_L} .$$

The input resistance is

$$R_i = r_b + r_c \left\{ \frac{r_e + R_L}{r_c(1-\alpha) + r_e + R_L} \right\}$$

Note that this is approximately $R_L/1-\alpha$ which is an order of magnitude larger than the emitter resistance R_L.

The power gain is $A^2\left(\frac{R_L}{R_i}\right)$ which is greater than unity because of the A^2 term.

The output resistance is given by

$$R_o = r_e + r_c (1-\alpha) \left\{ \frac{R_g + r_b}{R_g + r_b + r_c} \right\}$$

If we make the same assumptions as before, we obtain the following approximations:

$$G_v = 1$$

$$A = \frac{1}{1-\alpha} = \beta + 1$$

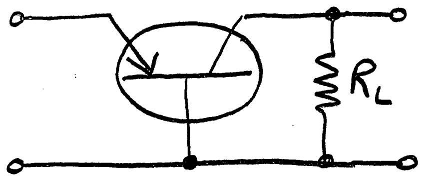
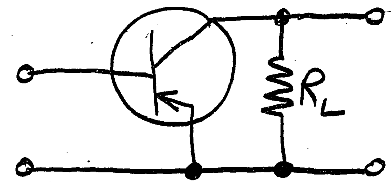
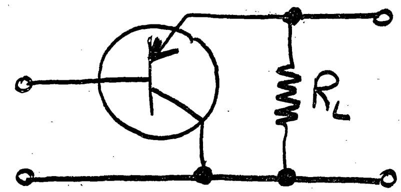
$$R_i = R_L/(1-\alpha) = R_L(\beta + 1)$$

$$P.G. = \frac{1}{1-\alpha} = \beta + 1$$

$$R_o = r_e + (r_b + R_g)(1-\alpha).$$

Note that the circuit has a relatively high input resistance and low output resistance.

A comparison of approximate formulae for the three configurations is given in Fig. 4.

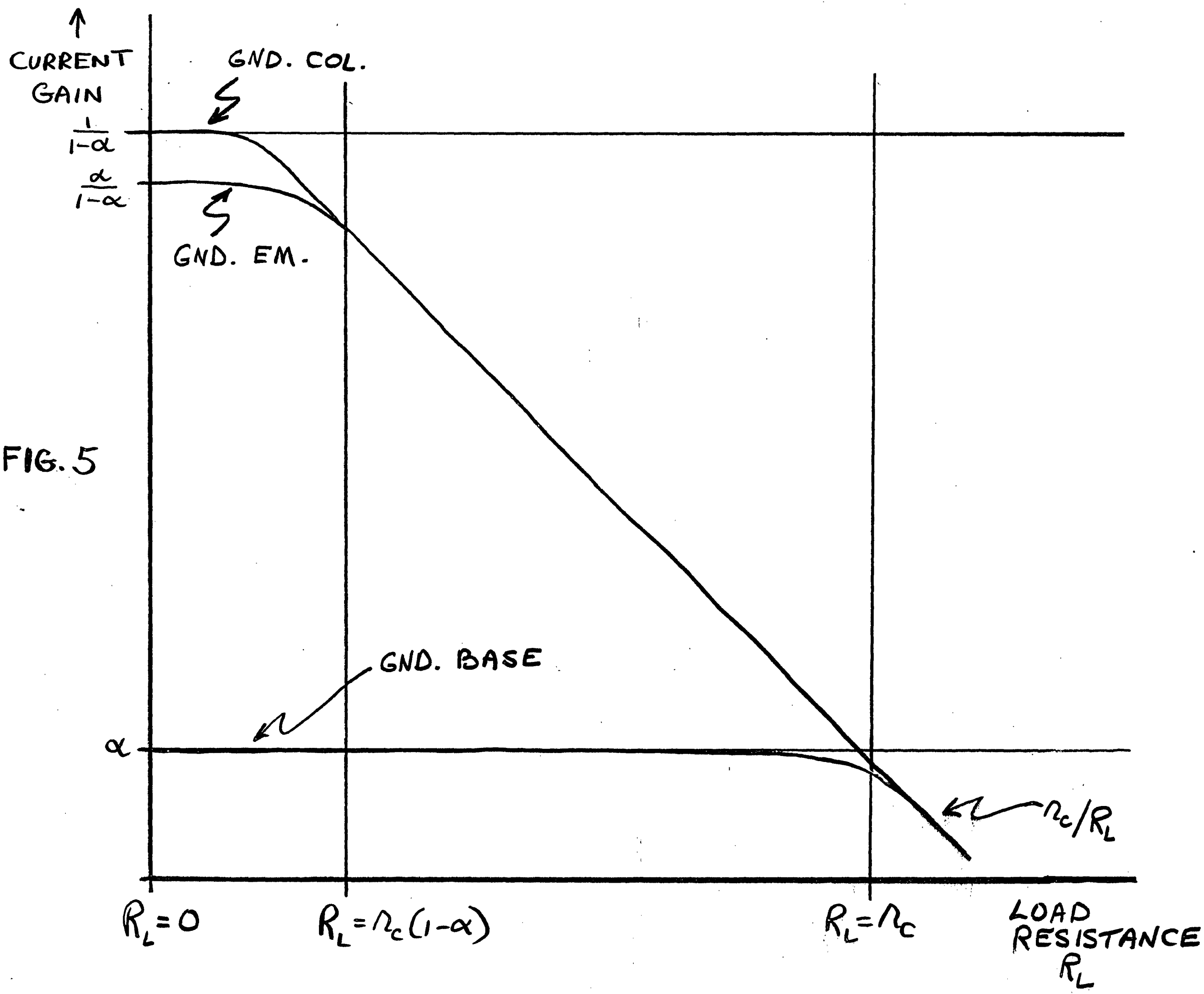
FIGURE 4 APPROX. FORMULAE ASSUMING $r_c(1-\alpha) \gg R_L \gg r_e, r_b$	Grounded-base 	Grounded-emitter 	Grounded-collector 
CURRENT GAIN	α	$\frac{\alpha}{1-\alpha} = \beta$	$\frac{1}{1-\alpha} = \beta + 1$
POWER GAIN	$\frac{\alpha^2 R_L}{r_e + r_b(1-\alpha)}$	$\left(\frac{1}{1-\alpha}\right)\left\{\frac{\alpha^2 R_L}{r_e + r_b(1-\alpha)}\right\}$	$\frac{1}{1-\alpha} = \beta + 1$
INPUT RESISTANCE	$r_e + r_b(1-\alpha)$	$r_b + \frac{r_e}{1-\alpha}$	$\frac{R_L}{1-\alpha} = (\beta + 1) R_L$
OUTPUT RESISTANCE	$r_c\left\{\frac{r_e + R_g + r_b(1-\alpha)}{r_e + R_g + r_b}\right\}$	$r_c(1-\alpha) + r_e\left\{\frac{\alpha r_c + R_g}{r_e + r_b + R_g}\right\}$	$r_e + (r_b + R_g)(1-\alpha)$
VOLTAGE GAIN	$\frac{\alpha R_L}{r_e + r_b(1-\alpha)}$	$\frac{-\alpha R_L}{r_e + r_b(1-\alpha)}$	1

4.0 Effect of Load Resistance on Amplifier Performance

In general we are not at liberty to change the internal transistor parameters given in the previous expressions. We can, however, vary the load R_L.

Fig. 5 shows the effect of load resistance on current gain for the 3 amplifiers.

FIG. 5



As R_L becomes greater than r_c all three have a current gain of about r_c/R_L.

The effect of load resistance on input resistance is shown in Figure. 6.

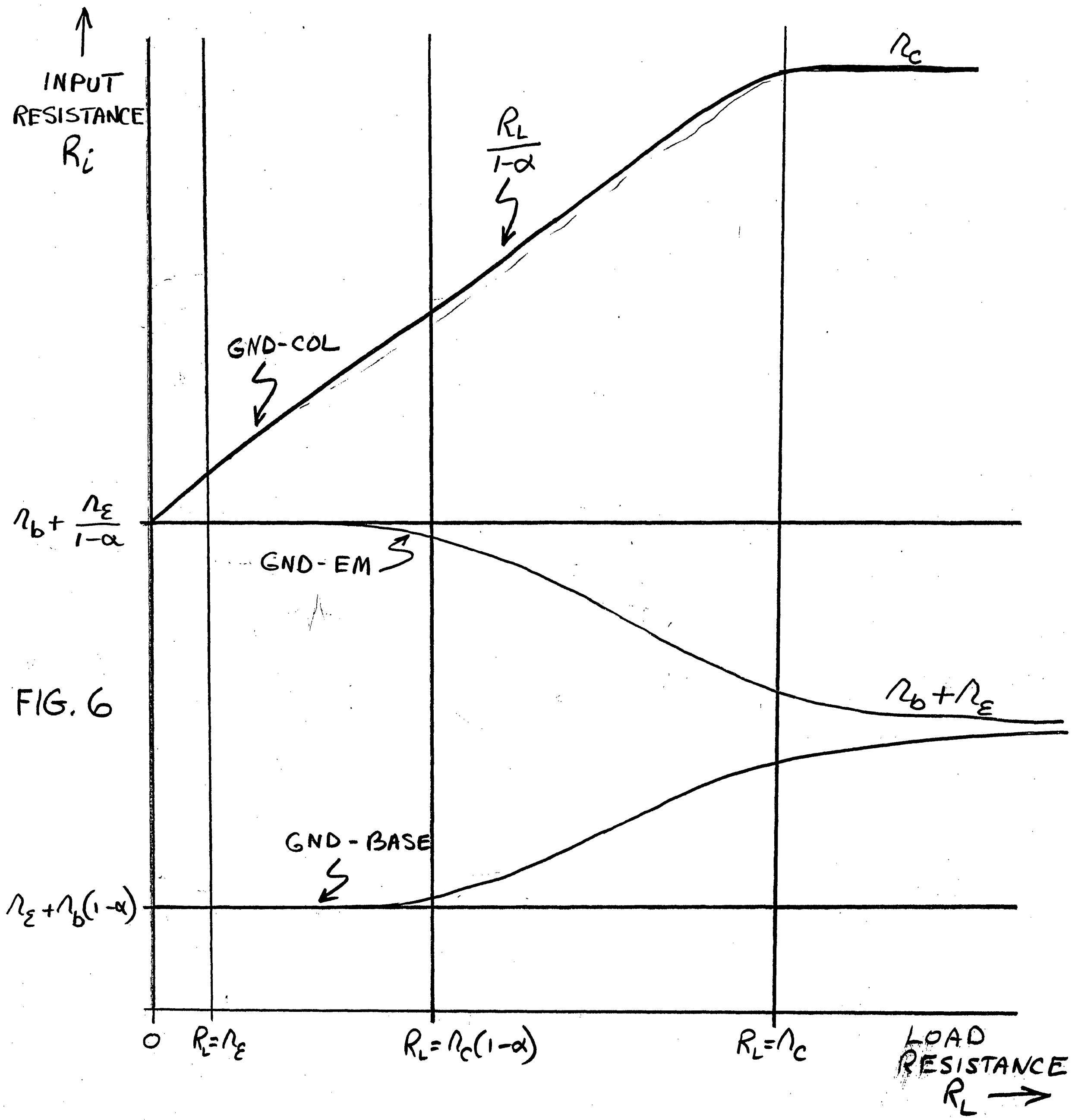


FIG. 6

The effect of the driving generator impedance on the output impedance of a transistor stage is shown in Figure 7. It is important to note the difference in current and voltage drive on the output impedance.

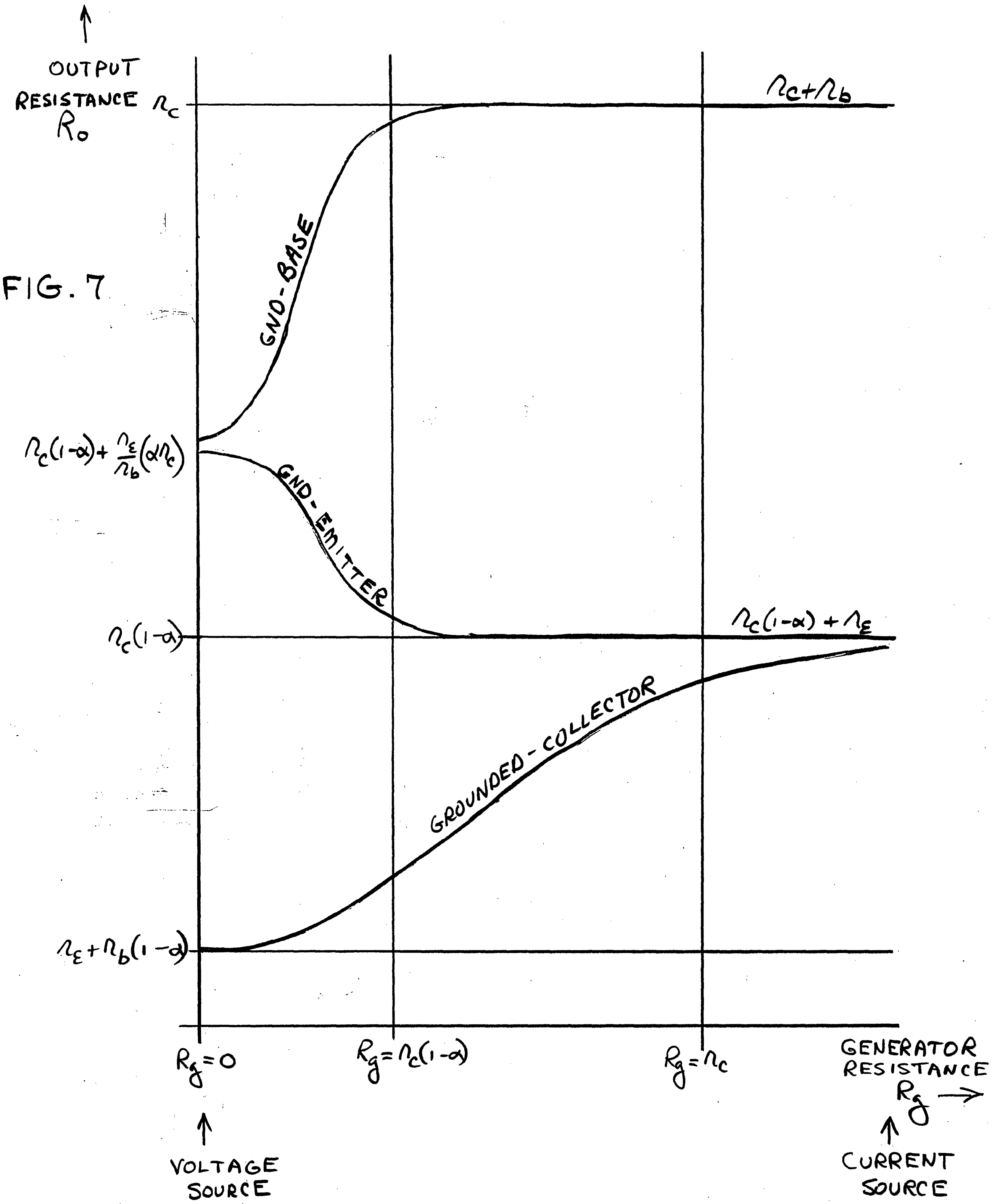

OUTPUT
RESISTANCE
R_o
r_c
r_c+r_b
GND-BASE
$r_c(1-\alpha)+\frac{r_\varepsilon}{r_b}(\alpha r_c)$
GND-EMITTER
$r_c(1-\alpha)$
$r_c(1-\alpha)+r_\varepsilon$
GROUNDED-COLLECTOR
$r_\varepsilon+r_b(1-\alpha)$
$R_g=0$
$R_g=r_c(1-\alpha)$
$R_g=r_c$
GENERATOR
RESISTANCE
R_g
VOLTAGE
SOURCE
CURRENT
SOURCE

FIG. 7

5.0 Frequency Response of Transistor Amplifiers

This is a subject which will be treated in more detail later on. The frequency response of the grounded-base amplifier is due to the variation in current-gain α with frequency. This can be expressed approximately as:

$$\alpha = \frac{\alpha_o}{1 + j\left(\frac{f}{f_{c\alpha}}\right)}$$

where α_o is the low frequency α and $f_{c\alpha}$ is the frequency at which α is $.707\alpha_o$. This expression is only an approximation and, in fact, one which is accurate only for $f/f_{c\alpha} \leq 1$.

The current gain of a grounded-emitter stage is

$$\beta = \frac{\alpha}{1-\alpha} = \frac{\dfrac{\alpha_o}{1 + j\left(\frac{f}{f_{c\alpha}}\right)}}{1 - \dfrac{\alpha_o}{1 + j\,(f/f_{c\alpha})}} = \frac{\alpha_o}{1 - \alpha_o + j\left(\frac{f}{f_{c\alpha}}\right)}$$

If we now divide top and bottom by $1 - \alpha_o$ we get

$$\frac{(\alpha_o/1 - \alpha_o)}{1 + j\,\frac{f}{f_{c\alpha}\,(1 - \alpha_o)}}$$

or $$\beta = \frac{\beta_o}{1 + j\,\frac{f}{f_{c\alpha}(1-\alpha_o)}}$$

Therefore the grounded-emitter stage has a frequency response $(1-\alpha_o)$ times that of the grounded-base circuit. This may be 1/10 or less.

The next chapter will discuss temperature stability of transistors.

Donald J. Eckl

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