Polynomial Curve Fitting Using Legendre Polynomials

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(fill out in typewriter, ink or pencil)

Program No._____ Date_____

Program Name:_____

1. Does the abstract adequately describe what the program is and what it does? Yes____ No____
   Comment_____

2. Does the program do what the abstract says? Yes____ No____
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3. Is the description clear, understandable, and adequate? Yes____ No____
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4. Are the Operating Instructions understandable and in sufficient detail? Yes____ No____
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   Are the Sense Switch options adequately described (if applicable)? Yes____ No____
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5. Does the source program compile satisfactorily (if applicable)? Yes____ No____
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6. Does the object program run satisfactorily? Yes____ No____
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7. Number of test cases run_____. Are any restrictions as to data, size, range, etc. covered adequately in description? Yes____ No____
   Comment_____

8. Does the Program meet the minimal standards of COMMON? Yes____ No____
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10. Please list on the back any suggestions to improve the usefulness of the program. These will be passed onto the author for his consideration.

Please return to:

Mr. Richard L. Pratt
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11/1/65

University of Notre Dame
Notre Dame, Indiana

Department of Civil Engineering

Explanation of Program

The program fits a polynomial of order 9 to a set of data points, using a trucated series of Legendre polynomials. While the running time is somewhat longer than a least squares fit, the Legendre series seems to follow any "bumps" in the data curve more closely, and several people have preferred this effect to the "smoothing out of the bumps" which the least squares fit tends to do.

One of the data input quantities needs a special explanation. A function f (x) can be represented by a Legendre series

$f(x) = \sum_{k=0}^{\infty} C_k P_k(x)$, $C_k = \frac{2k+1}{2} \int_{-1}^{1} f(x) P_k(x)\, dx$, where $P_k(x)$ are Legendre polynomials. The higher order polynomials oscillate badly in the interval (- 1, 1), so the interval length chosen for the integral for C_k must be small, in many cases many times smaller than the intervals between the given data points. Thus a quantity IJ is introduced to represent the number of subintervals between adjoining data points. Then f (x) is interpolated linearly at the ends of each subinterval.

To take an example there should be a total of around 200 subintervals in the interval (- 1, 1) for a reasonably good integration of $P_9(x)\, f(x)$. If there are, say, 12 data points, IJ should be about 20.

Modifications or revisions to this program, as they occur, will be announced in the appropriate Catalog of Programs for IBM Data Processing Systems. When such an announcement occurs, users should order a complete new program from the Program Information Department.

DATA INPUT FORMAT

Punched Cards. Order of Data: N (Number of Data Points) IJ (Number of Subintervals Between Adjacent Data Points).

X_1, $F(X_1)$, X_2, $F(X_2)$, ..., X_n, $F(X_n)$, as closely as one can pack them on a card or cards.

DATA OUTPUT FORMAT

Punched Cards.

If SSW1 is on, the Legendre series coefficients are punched.

If SSW2 is on, the coefficients of the power series $f(x) \sum_{k=1}^{10} = P_k\, w^{k-1}$ are punched.

If Sense Switch 3 is on, the x values are "normalized" into the interval (-1, 1), and these quantities are punched: x, w, s, d, for each data point,

where, x = abscissa of data point

w = normalized value of x

s = value of truncated Legendre series

d = difference between s and f, the ordinate of data point

If Sense Switch 3 is off, the Legendre series in w is converted to a power series in x, and these quantities are punched:

the coefficients r_k of the power series $f(x) = \sum_{k=1}^{10} r_k x^{k-1}$

x, s, d for each data point, where x, s d are as defined above.

MACHINE CONFIGURATION

IBM 1620 with card reader- punch 60 K memory.

PROGRAM INSTRUCTIONS

Beginning of program: 07500

Load card reader, push start.

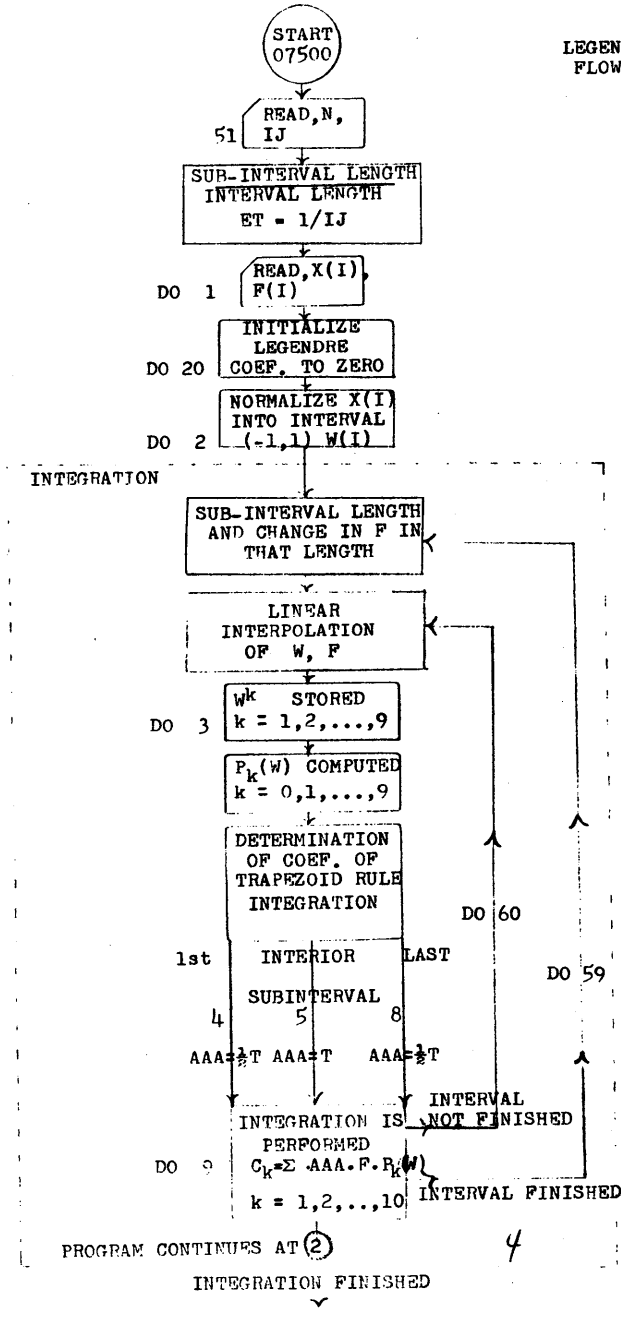
If Legendre series coefficients are desired, put Sense Switch 1 on. If normalized power series coefficients are desired, put Sense Switch 2 on.

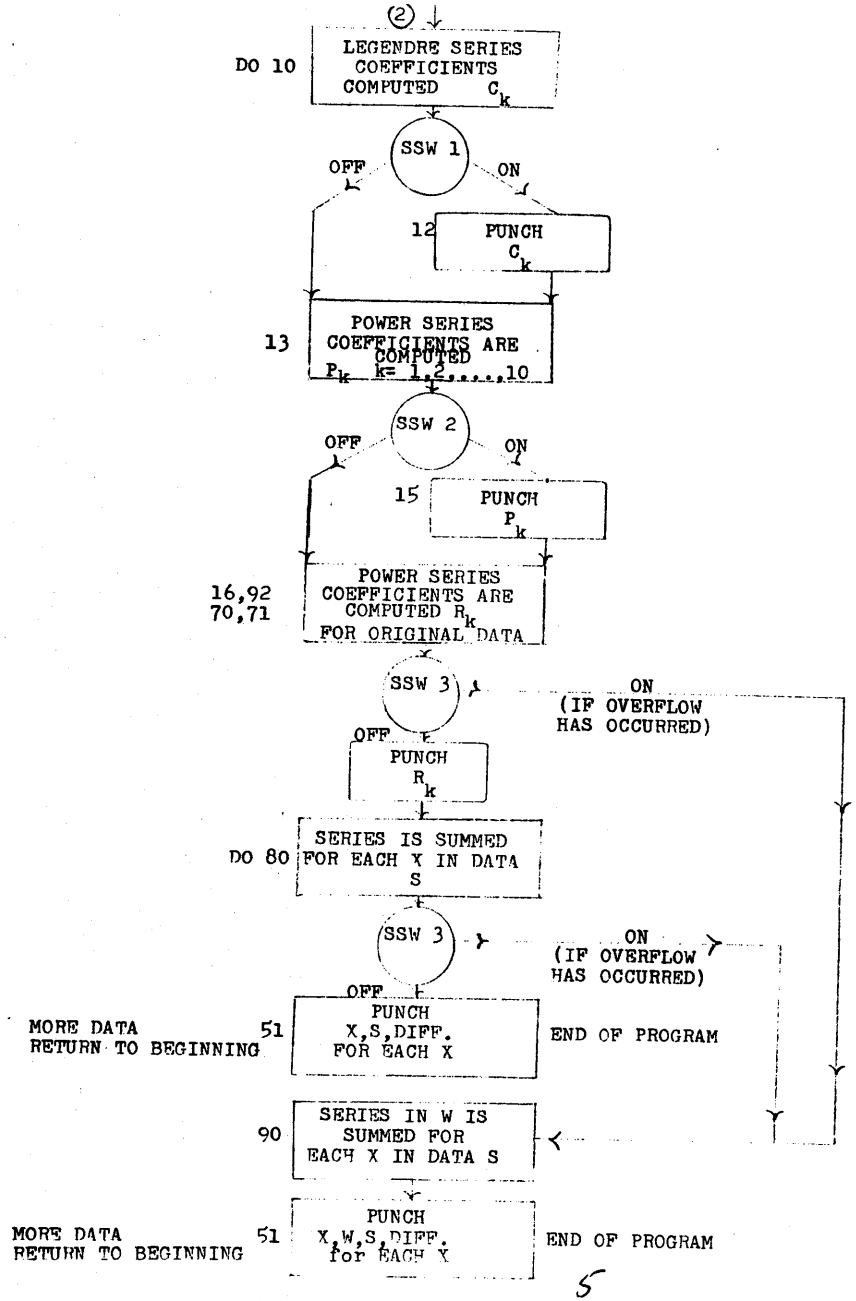
If summation of normalized power series is desired, put Sense Switch 3 on. This may be necessary, because the regular power series is liable to overflow if abscissas are large.

When program is finished, new data is called for.

Program running time in seconds: T ≈ 1.5 x (Number of Data Points) x (Number of Subintervals between data points) = 1.5 x (N) x (IJ) seconds.

LEGENDRE FIT
FLOW CHART





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POLYNOMIAL CURVE FITTING USING LEGENDRE POLYNOMIALS

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   DIMENSION X(100),F(100),C(10),W(100),V(9),P(10),A(9),B(9),R(10)
51 READ,N,IJ
   ET=IJ
   ET=1.0000000/ET
   IJ=IJ+1
   DO 1 I=1,N
   READ,X(I),F(I)
 1 CONTINUE
   DO 20 J=1,10
   C(J)=.00000000
20 CONTINUE
   DO 2 I=1,N
   W(I)=2.0000000*(X(I)-X(1))/(X(N)-X(1))-1.0000000
 2 CONTINUE
   M=N-1
   DO 59 I=1,M
   Q=.00000000
   T=ET*(W(I+1)-W(I))
   TT=ET*(F(I+1)-F(I))
   DO 60 J=1,IJ
   V(1)=W(I)+T*Q
   FF=F(I)+TT*Q
   Q=Q+1.0000000
   DO 3 K=2,9
   V(K)=V(K-1)*V(1)
 3 CONTINUE
   P(1)=1.0000000
   P(2)=V(1)
   P(3)=1.5000000*V(2)-.50000000
   P(4)=2.5000000*V(3)-1.5000000*V(1)
   P(5)=4.3750000*V(4)-3.7500000*V(2)+.37500000
   P(6)=7.8750000*V(5)-8.7500000*V(3)+1.8750000*V(1)
   P(7)=14.437500*V(6)-19.687500*V(4)+6.5625000*V(2)-.31250000
   P(8)=26.812500*V(7)-43.312500*V(5)+19.687500*V(3)-2.1875000*V(1)
   P(9)=50.273438*V(8)-93.843750*V(6)+54.140625*V(4)-9.8437500*V(2)
   P(9)=P(9)+.27343750
   P(10)=94.960938*V(9)-201.09375*V(7)+140.76562*V(5)-36.093750*V(3)
   P(10)=P(10)+2.4609375*V(1)
   IF (J-2) 4,5,5
 4 AAA=.50000000*T
   GO TO 7
 5 AAA=T
 6 IF (J-IJ)7,8,8
 8 AAA=.50000000*T
 7 DO 9 K=1,10
   C(K)=C(K)+AAA*FF*P(K)
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 9 CONTINUE
60 CONTINUE
59 CONTINUE
   DO 10 K=1,10
   E=K-1
   C(K)=.50000000*C(K)*(2.0000000*E+1.0000000)
10 CONTINUE
   IF(SENSE SWITCH 1)12,13
12 PUNCH,C(1),C(2),C(3),C(4)
   PUNCH,C(5),C(6),C(7),C(8)
   PUNCH,C(9),C(10)
13 P(10)=94.960938*C(10)
   P(9)=50.273438*C(9)
   P(8)=26.812500*C(8)-201.09380*C(10)
   P(7)=14.437500*C(7)-93.843750*C(9)
   P(6)=7.8750000*C(6)-43.312500*C(8)+140.76562*C(10)
   P(5)=4.3750000*C(5)-19.687500*C(7)+54.140625*C(9)
   P(4)=2.5000000*C(4)-8.7500000*C(6)+19.687500*C(8)-36.093750*C(10)
   P(3)=1.5000000*C(3)-3.7500000*C(5)+6.5625000*C(7)-9.8437500*C(9)
   P(2)=C(2)-1.5000000*C(4)+1.8750000*C(6)-2.1875000*C(8)
   P(2)=P(2)+2.4609375*C(10)
   P(1)=C(1)-.50000000*C(3)+.37500000*C(5)-.31250000*C(7)
   P(1)=P(1)+.27343750*C(9)
   IF(SENSE SWITCH 2)15,16
15 PUNCH,P(1),P(2),P(3),P(4)
   PUNCH,P(5),P(6),P(7),P(8)
   PUNCH,P(9),P(10)
16 AA=2.0000000/(X(N)-X(1))
   BB=-AA*X(1)-1.0000000
   A(1)=AA
   DO 70 I=2,9
   A(I)=AA*A(I-1)
70 CONTINUE
   B(1)=BB
   DO 71 I=2,9
   B(I)=BB*B(I-1)
71 CONTINUE
   IF(SENSE SWITCH 3)90,92
92 R(1)=P(1)+P(2)*B(1)+P(3)*B(2)+P(4)*B(3)+P(5)*B(4)+P(6)*B(5)
   R(1)=R(1)+P(7)*B(6)+P(8)*B(7)+P(9)*B(8)+P(10)*B(9)
   R(2)=P(2)+2.0000000*P(3)*B(1)+3.0000000*P(4)*B(2)
   R(2)=R(2)+4.0000000*P(5)*B(3)+5.0000000*P(6)*B(4)
   R(2)=R(2)+6.0000000*P(7)*B(5)+7.0000000*P(8)*B(6)
   R(2)=A(1)*(R(2)+8.0000000*P(9)*B(7)+9.0000000*P(10)*B(8))
   R(3)=P(3)+3.0000000*P(4)*B(1)+6.0000000*P(5)*B(2)
   R(3)=R(3)+10.000000*P(6)*B(3)+15.000000*P(7)*B(4)
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   R(3)=R(3)+21.000000*P(8)*B(5)+28.000000*P(9)*B(6)
   R(3)=A(2)*(R(3)+36.000000*P(10)*B(7))
   R(4)=P(4)+4.0000000*P(5)*B(1)+10.000000*P(6)*B(2)
   R(4)=R(4)+20.000000*P(7)*B(3)+35.000000*P(8)*B(4)
   R(4)=A(3)*(R(4)+56.000000*P(9)*B(5)+84.000000*P(10)*B(6))
   R(5)=P(5)+5.0000000*P(6)*B(1)+15.000000*P(7)*B(2)
   R(5)=R(5)+35.000000*P(8)*B(3)+70.000000*P(9)*B(4)
   R(5)=A(4)*(R(5)+126.00000*P(10)*B(5))
   R(6)=P(6)+6.0000000*P(7)*B(1)+21.000000*P(8)*B(2)
   R(6)=A(5)*(R(6)+56.000000*P(9)*B(3)+126.00000*P(10)*B(4))
   R(7)=P(7)+7.0000000*P(8)*B(1)+28.000000*P(9)*B(2)
   R(7)=A(6)*(R(7)+84.000000*P(10)*B(3))
   R(8)=A(7)*(P(8)+8.0000000*P(9)*B(1)+36.000000*P(10)*B(2))
   R(9)=A(8)*(P(9)+9.0000000*P(10)*B(1))
   R(10)=A(9)*P(10)
   PUNCH,R(1),R(2),R(3),R(4)
   PUNCH,R(5),R(6),R(7),R(8)
   PUNCH,R(9),R(10)
   IF(SENSE SWITCH 3)90,93
93 DO 80 I=1,N
   V(1)=X(I)
   DO 81 J=2,9
   V(J)=V(J-1)*V(1)
   IF(SENSE SWITCH 3)90,81
81 CONTINUE
   S=R(1)
   DO 82 J=1,9
   S=S+R(J+1)*V(J)
   IF(SENSE SWITCH 3)90,82
82 CONTINUE
   D=S-F(I)
   PUNCH,X(I),S,D
80 CONTINUE
   GO TO 51
90 DO 17 I=1,N
   V(1)=W(I)
   DO 18 J=2,9
   V(J)=V(J-1)*V(1)
18 CONTINUE
   S=P(1)
   DO 19 J=1,9
   S=S+P(J+1)*V(J)
19 CONTINUE
   D=S-F(I)
   PUNCH,X(I),W(I),S,D
17 CONTINUE
   GO TO 51
   END
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